Zhou, Z., X. Li, and B. Liu. *Bethe's Calculation For Solar Energy And Selective Resonant Tunneling*. in *Tenth International Conference on Cold Fusion*. 2003. Cambridge, MA: LENR-CANR.org. This paper was presented at the 10th International Conference on Cold Fusion. It may be different from the version published by World Scientific, Inc (2003) in the official Proceedings of the conference.

BETHE'S CALCULATION FOR SOLAR ENERGY AND SELECTIVE RESONANT TUNNELING

XIANG ZHOU , XING Z. LI[1] ,BIN LIU[1]

Department of Engineering Physics, Tsinghua University, Beijing 100084, CHINA

[1]Department of Physics, Tsinghua University, Beijing 100084, CHINA

lxz-dmp@tsinghua.edu.cn

The Selective Resonant tunneling model is compared with Bethe's early model for the solar energy calculation. They are similar in considering the resonance effect, the weak interaction, and the assumption for nuclear potential and the Coulomb barrier in order to obtain the correct result for the energy density in the sun. However, the selectivity of resonant tunneling is new in the present selective resonant model.

1 Introduction

In 2000, Dr. Roberto Andreani made a speech on closing session of the ICCF-8 (Lerice, Italy) that fusion energy is not necessary accompanied with the neutron radiation. He said that the solar energy was originated from proton-proton fusion, but there was no neutron accompanied. R. Andreani was the deputy director of the hot fusion project in ENEA (Frascati), Italy. He was leaving for Garching, Germany to work as the deputy director of the European team for ITER project. However, he paid attention to the cold fusion project from the beginning, and he did not refuse the cold fusion. Radiation-less fusion energy is a strong point of "cold fusion". There was no reason to discredit the "cold fusion" just based on lack of neutron radiation. Selective resonant tunneling model was proposed in 1995 to explain why the fusion reaction should not be accompanied with neutron radiation. It is interesting to investigate how Bethe obtained his solar energy density in 1938, which was one of his merits to win the Nobel Prize later in 1967[1]

2 Bethe's Calculation of the Solar Energy Density

2.1 Resonance Effect

Bethe clearly mentioned resonance twice in his paper [1], and the contribution from the resonance was more than one half of the total fusion cross section (see Table I in [1]). It is understandable, because Bethe believed that the temperature of solar plasma was 2 keV only which was much lower than the height of the Coulomb barrier between two protons. The quantum mechanics tunneling without resonance might not be enough to provide the solar energy density in order to meet the observation in astrophysics.

Bethe assumed a square-well for nuclear interaction, and a pure Coulomb potential outside the nuclear interaction region between two protons. Hence, three parameters would be needed in order to determine the wave function of the relative motion of two protons in their center of mass system. These parameters were the depth and the radius of the nuclear potential,(D and r_0, respectively), and the kinetic energy of the incident proton (E). In Bethe's terminology, resonance just meant the mixture of the regular Coulomb wave function (F_0) with the irregular Coulomb wave function (G_0). The ratio of the mixture was determine by the phase shift of the Coulomb wave function (δ_0). Here the subscript 0 just means the s-partial wave of the wave function. This phase shift, δ_0, might be calculated when the abovementioned three parameters were fixed:

$$Tan\delta_0 = \frac{F_0^2\kappa_0}{1 - F_0 G_0 \kappa_0}, \tag{1}$$

$$\kappa_0 = \left[\frac{dLogF_0}{d\rho} - \frac{dLogw}{d\rho}\right]_{r_0}, \tag{2}$$

$$w = Sink_1 r\,. \tag{3}$$

where, w is proportional to the reduced radial wave function inside the nuclear well.

$$k_1 = \sqrt{\frac{2\mu}{\hbar^2}(E + D)} \approx \sqrt{\frac{2\mu D}{\hbar^2}}, \tag{4}$$

Here, $\rho = kr$, μ is the reduced mass for 2 protons, $k = \sqrt{\frac{2\mu E}{\hbar^2}}$. The wave function outside the nuclear well were written as the summation of the F_0 and G_0 .i.e.

$$\psi_p \propto \frac{(F_0 + Tan\delta_0 G_0)}{kr}, \tag{5}$$

When energy E is low, and r_0 is small, Bethe used the approximation that

$$F_0 = C\rho\,\Phi, \quad G_0 = \frac{\Theta}{C}.$$

$$\Phi(y) = \frac{I_1(2\sqrt{y})}{\sqrt{y}} = 1 + \frac{y}{1!2!} + \frac{y^2}{2!3!} + \frac{y^3}{3!4!} + \ldots\ldots \tag{6}$$

$$\Theta(y) = -2\sqrt{y}K_1(2\sqrt{y}) = 1 + y(Logy + 2\gamma - 1)\,\Phi - \sum_{s=1}^{\infty}\frac{y^{s+1}}{s!(s+1)!}\sum_{t=1}^{s}\left(\frac{1}{t} + \frac{1}{t+1}\right)$$

$$y = 2r/a_c,\; a_c = \frac{\hbar^2}{\mu e^2} = 5.75\times 10^{-12}\,cm,\; \gamma = 0.577\ldots\text{(Euler's constant)}.$$

Hence,

$$Tan\delta_0 = C^2 k r_0 \lambda \quad , \tag{7}$$

$$\lambda = \frac{\Phi^2\varsigma}{1 - \Phi\Theta\varsigma} \quad , \tag{8}$$

$$\varsigma = \left[\frac{dLog(F_0/w)}{dLog\,\rho}\right]_{r_0} \quad , \tag{9}$$

$$C = \sqrt{2\pi\eta}Exp[-\pi\eta], \quad \eta = \frac{e^2}{\hbar v}, \tag{10}$$

Here, v is the relative speed between two protons. Under the Bethe's approximation, ζ is independent of the incident energy, E. Φ and Θ approach 1 when $r \to 0$. $Tan\delta_0$ depends on energy E through C^2kr_0 only, which is quite small due to the Coulomb barrier and the small value of k. This dependence will have a Gamow peak when the Maxwell velocity distribution of protons was considered. However, it is interesting to notice that even if the $Tan\delta_0 = 0.0017$ at this Gamow peak, the contribution of this G_0 to the fusion cross section is more than 50% in the final expression for the total cross section (see Table I later). We may see that resonance is an important factor in the low energy fusion reaction. Particularly, when the nuclear potential well was assumed such that $Tan\delta_0 >> 1$; then, the resonance effect would reach its maximum value, and dominate the fusion cross section.

2.2 Weak Interaction

Bethe clearly stated that weak interaction played key role for the fusion reactions in the sun. Four protons would be eventually combined together to form an alpha-particle and emit two positrons. The energy released is 26.8 MeV. Although a chain of reactions might be involved in the whole process, the first reaction was the formation of deuterons by proton combination, i.e.

$$p + p \rightarrow d + e^{+} + \nu_e \tag{11}$$

The Fermi constant for this weak interaction was a very small number g:

$$g = 0.9\times10^{-4}\ \text{sec}^{-1} \tag{12}$$

hence, the perturbation theory was applied to calculate the transition probability from the 2 protons to the ground state of deuteron. The cross section for the combination of two protons of relative velocity v was given by

$$\sigma = \frac{g\ f(W)}{v}\left|\int \psi_p \psi_d \, d\tau\right|^2 \tag{13}$$

Here, $f(W)$=0.132(the Coulomb correction factor in weak interaction), and

$$\left|\int \psi_p \psi_d \, d\tau\right|^2 = C^2 \, \frac{(4\pi b^2)^2}{2\pi b} \, \frac{(\Lambda_1 + \Lambda_2 + \Lambda_3)^2}{(1+x_0)(1+\mu^{-2})} \tag{14}$$

Here, $b=4.37\times10^{-13}$ cm is about the radius of the deuteron, $x_0=r_0/b$. Λ_1 is the integration inside the nuclear well; both Λ_2 and Λ_3 are the integration outside the nuclear well. Λ_2 is the integration which involver Φ; Λ_3 is the integration which involves Θ. Hence, Λ_3 represents the resonance effect. Based on the Table I in Bethe's paper[1], we calculated the contribution from Λ_3 which is more than 50% for his sets of parameters.(see Table I below).

Table I. Contribution from Resonance

	$r_0=e^2/(mc^2)$	$r_0=e^2/(2mc^2)$
Λ_1	0.689	0.277
Λ_2	1.949	1.547
Λ_3	1.205	1.030
$[(\Lambda_1+\Lambda_2)/(\Lambda_1+\Lambda_2+\Lambda_3)]^2$	0.471	0.408
Contribution from Resonance (Λ_3)	**53%**	**59%**

3 Selectivity of the Resonant Tunneling

From the abovementioned discussion, we can see that Bethe's calculation did not consider the selectivity of the resonant tunneling, because Bethe did not consider the effect of the absorption on the wave function. The phase shift, $Tan\delta_0$, was a real number in Bethe's calculation. Indeed the phase shift should be a complex number in case of fusion reaction. Consequently, the tunneling probability in Bethe's model did not show any dependence on energy other than Gamow factor ($2\pi\eta\ \exp[-2\pi\eta]$) and geometric factor ($1/k^2$); therefore, there was no selectivity in the resonant tunneling effect in Bethe's approximation.

In Bethe's approximation, the dependence of cross section of fusion reaction on the energy appeared only in C^2/v. Two dependences on energy are neglected. The first is the k_1, which is the wave number inside the nuclear well. When nuclear well is deep, and the incident energy is low, $|E/D| \ll 1$; hence, k_1 is almost independent on E. The second is the dependence of Θ on E. In Landau's approximation [2,3],

$$\Theta = 1 + y\{Log\ y + 2\gamma - 1 + h(ka_c)\} \tag{15}$$

Here,

$$h(ka_c) \equiv \mathrm{Re}\{\frac{\Gamma'(\frac{-i}{ka_c})}{\Gamma(\frac{-i}{ka_c})}\} + Log(ka_c) \approx \frac{(ka_c)^2}{12}, \quad \text{when } (ka_c) << 1. \tag{16}$$

This weak dependence on (ka_c) is important for the selectivity of resonant tunneling also.

In the selective resonant tunneling model, an imaginary potential for nuclear well was introduced to make the phase shift complex such that[3]

$$\sigma = \frac{\pi}{k^2}\frac{-4\,\mathrm{Im}[Cot\delta_0]}{\{\mathrm{Re}[Cot\delta_0]\}^2 + \{\mathrm{Im}[Cot\delta_0]-1\}^2} = \frac{\pi}{k^2}C^2kr_0\frac{-4\,\mathrm{Im}[\frac{1}{\lambda}]}{\{\mathrm{Re}[\frac{1}{\lambda}]\}^2 + \{\mathrm{Im}[\frac{1}{\lambda}]-C^2kr_0\}^2} \tag{17}$$

The complex nuclear potential makes λ complex, and those abovementioned two dependences on energy make

$$\left\{\begin{matrix} \mathrm{Re}[\frac{1}{\lambda}] = 0 \\ \mathrm{Im}[\frac{1}{\lambda}] = -C^2kr_0 \end{matrix}\right\} \Rightarrow \sigma = \frac{\pi}{k^2}\,. \tag{18}$$

This feature had been ignored in Bethe's calculation.

3 Concluding Remarks

From Bethe's early work[1] we may conclude that:

(1) Resonance effect is necessary to obtain the correct number for the energy density in the sun;

(2) Weak interaction gives correct order of magnitude for fusion reaction in the sun, although it is very weak;

(3) Resonant tunneling effect in Bethe's calculation was not complete.

Acknowledgements

This work is supported by The Ministry of Science and Technology (Fundamental Division), Natural Science Foundation of China (#10145004) and Tsinghua University (Basic Research Fund (985-I)).

References

[1] H. A. Bethe, and C. L. Critchfield, "The Formation of Deuterons by Proton Combination", Phys. Rev. **54** 248 (1938).

[2] L. D. Landau and E. M. Lifshitz, "Quantum Mechanics,"(Pergamon Press, Oxford,1987), p.571.

[3] Xing Z. Li, "Nuclear Physics for Nuclear Fusion". *Fusion Science and Technology*, **41** 63 (2002).