ISSN 1063-7761, Journal of Experimental and Theoretical Physics, 2015, Vol. 121, No. 4, pp. 559–571. © Pleiades Publishing, Inc., 2015.
Original Russian Text © V.I. Vysotskii, M.V. Vysotskyy, 2015, published in Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki, 2015, Vol. 148, No. 4, pp. 643–657.

ATOMS, MOLECULES, OPTICS

The Formation of Correlated States and Optimization of the Tunnel Effect for Low-Energy Particles under Nonmonochromatic and Pulsed Action on a Potential Barrier

V. I. Vysotskii and M. V. Vysotskyy
Taras Shevchenko National University, Kiev, 01601 Ukraine
e-mail: vivysotskii@gmail.com
Received March 5, 2015

Abstract—We consider peculiarities of the formation of a coherent correlated state (CCS) of a low-energy particle under frequency modulation of parameters of a harmonic oscillator that contains this particle by a broadband nonmonochromatic or asymmetric pulsed action. It is shown that in the case of modulation with frequency-normalized intensity, the maximum efficiency of CCS formation corresponds to a narrow-band action, while broadband modulation is optimal for the action with a constant spectral density. As in the case of monochromatic modulation, the maximum correlation coefficient, $|r|_{\max}$, under the nonmonochromatic action corresponds to parametric resonance at frequency $\Omega \approx 2\omega_0$. Under a pulsed action, the maximum efficiency of CCS formation and, hence, the maximum probability of the tunnel effect, correspond to pulsed modulation with a short leading edge and a long trailing edge. In particular, under the action of a pulsed magnetic field with an amplitude of 10 kOe and the leading edge duration of 2×10^{-7} s on a gas with deuterium ions, a CCS can be formed with the correlation coefficient $|r|_{\max} \approx 0.9998$, for which the tunneling effect probability under the *dd* interaction at temperature $T \approx 300$–500 K increases from $D_{r=0} \approx 10^{-80}$ to $D_{|r|_{\max}=0.9998} \approx 0.1$. This process can occur in a gas with particle number density $n < n_{\rm cr} \approx 10^{17}$ cm^{-3}. The method of CCS formation makes it possible to explain the results of an experiment in which substantial isotope changes were detected when a pulsed electric current and magnetic-field generation occurred.

DOI: 10.1134/S1063776115100222

1. INTRODUCTION

One of the most important problems in applied and fundamental physics is the occurrence of nuclear reactions (including nuclear fusion) at low energies. This problem is especially topical due to successful experiments (in particular, those performed in Lugano [1]), in which fundamental nuclear transformations with a radical change in the isotope composition of an active medium were observed under the conditions that were far from the stringent requirements for thermonuclear fusion. These processes occur at a low energy and cannot be explained proceeding from the "conventional" tunnel effect, which predicts a very low tunneling probability of $D \leqslant 10^{-100}$.

It should be noted that the tunnel effect is based on the fruitful idea of wave–particle duality, which was first represented in the form of the Heisenberg uncertainty relationship $\delta q \delta p \geqslant \hbar/2$ and was subsequently generalized in the form of the Heisenberg–Robertson uncertainty relationship

$$\sigma_A \sigma_B \geq |\langle[\hat{A}\hat{B}]\rangle|^2/4, \tag{1}$$

which makes it possible to determine the limitations that are imposed on the product of dispersions of two dynamic variables, A and B. These relationships correspond to the so-called uncorrelated states of the particle.

Subsequent more specific investigations that were carried out by Schrödinger [2] and Robertson [3] have shown that the use of specially formed coherent correlated states (CCSs) of particles leads to a modified Schrödinger–Robertson uncertainty relationship

$$\begin{gathered} \sigma_A \sigma_B \geq |\langle[\hat{A}\hat{B}]\rangle|^2/4(1-r^2), \\ r = \sigma_{AB}/\sqrt{\sigma_A \sigma_B}, \\ \sigma_{AB} = (\langle \hat{A}\hat{B} + \hat{B}\hat{A}\rangle)/2 - \langle A\rangle\langle B\rangle, \quad |r| \leq 1. \end{gathered} \tag{2}$$

It should be noted that the term CCS as applied to this uncertainty relationship was used for the first time in [4].

Subsequent investigations [4–16] showed that the application of CCSs can lead to significant optimization of tunneling probability $D_{|r| \to 1} \to 1$. Here, r is the correlation coefficient determining the interrelation between quantities A and B.

The uncorrelated state corresponds to $r = 0$, while the completely correlated state corresponds to $|r| = 1$. The effect of CCS for $|r| \to 1$ is characterized by the correlation effectiveness coefficient $G \equiv 1/\sqrt{1-r^2}$ [13, 16, 17], which varies in the interval $1 \leqslant G < \infty$ and determines the enhancement of fluctuations of A and B.

When $A = q$, $B = p$, $\langle q\rangle = 0$, $\langle p\rangle = 0$, and $\sigma_q = \langle q^2\rangle$, $\sigma_p = \langle p^2\rangle$ relationships (2) have the form

$$\sigma_q\sigma_p \geq \hbar^2/4(1-r^2) \equiv G^2\hbar^2/4. \quad (3)$$

This relationship can be used for obtaining a simple estimate that visually demonstrates the effectiveness of the application of a CCS for increasing the tunneling effect probability. If we consider a particle (e.g., a proton with mass M_p) in an interatomic potential well of width $L \approx \sqrt{\sigma_q} \approx 10^{-8}$ cm, the kinetic energy fluctuation in an uncorrelated state with $r = 0$ is limited by the value $\delta T_{r=0} = \sigma_p/2M \geqslant \hbar^2/8Ma^2 \approx 0.05$ eV. Obviously, the tunneling probability for such energy is negligibly low ($D_{r=0} \to 0$).

In the case of CCS formation with the attainable value of the correlation coefficient $1 - |r| \approx 10^{-6}$, the fluctuation energy increases sharply to a very large value $\delta T_{|r|=1-10^{-6}} = \delta T_{r=0}G^2 \geqslant 25$ keV, leading to effective tunneling.

It should be noted that the above estimates for energy fluctuation $\delta T_{r \neq 0}$ determine only the lower threshold. Rigorous quantum-mechanical calculation of the wavefunction in the classically forbidden region [8–11] makes it possible to state with confidence that the value of such a fluctuation can be much higher. The possibilities of further modification of the uncertainty relationships were considered in [18].

The existence of correlation can be taken into account in obtaining qualitative estimates using the formal substitution $\hbar \to \hbar^* \equiv \hbar/\sqrt{1-r^2} \equiv G\hbar$ in the expression for D [7]. This substitution in some cases (e.g., for localization of a particle in a parabolic well [11–13]) corresponds to the approximate formula for the tunneling probability in subbarrier region $L(E)$ in the nuclear field of radius R:

$$D_{r\neq 0} \approx \exp\left\{-\frac{2\sqrt{1-r^2}}{\hbar}\int\limits_{R}^{R+L(E)}\sqrt{2M\{V(q)-E\}}dq\right\} = (D_{r=0})^{\sqrt{1-r^2}} \equiv \sqrt[G]{D_{r=0}} \quad (4)$$

and is in good agreement with the results of an independent rigorous calculation of $D(r)$ based on the criterion $|\ln D(r) - \ln D_{r\neq 0}(r)|/|\ln D(r)| \ll 1$ for a small initial value of $D_{r=0} \ll 1$ [8].

In [14, 15], the problem of the passage of a narrow Gaussian wave packet through a model delta barrier and an actual Coulomb barrier was considered; it was shown that the tunneling probability in this case coincides qualitatively with expression (4) and differs in the replacement of the exponent $\sqrt{1-r^2}$ on the right-hand side by quantity $\sqrt[3]{1-r^2}$.

A physical model that substantiates the possibility of giant fluctuations of the particle energy in the CCS and is associated with synchronization of fluctuations of momentum in the multilevel superposition state was considered in [11, 12].

It should be noted that the generalized Schrödinger–Robertson uncertainty relationship and the CCS concept were successfully used in analysis of problems not associated with optimization of the tunnel effect (in particular, in analysis of the model of quantum-mechanical Brownian motion [19] or in analysis of peculiarities of diffusion of quantum states [20]).

2. GENERAL PROBLEMS AND METHODS OF CCS FORMATION IN NONSTATIONARY SYSTEMS

Detailed descriptions of the two main regimes of CCS formation based on the basic model of a harmonic oscillator were given in [4–13, 16]. The computation aspect of this model corresponds to the solution of the time-dependent Schrödinger equation for various regimes of periodic or monotonic deformation of harmonic potential $V(q, t) = M\omega^2(t)q^2/2$, in the field of which the particle is located.

The solution of this equation shows that the explicit form of the correlation coefficient

$$r = \mathrm{Re}\left\{\varepsilon^*\frac{d\varepsilon}{dt}\right\}/\left|\varepsilon^*\frac{d\varepsilon}{dt}\right|, \quad r^2 = 1 - \left|\varepsilon^*\frac{d\varepsilon}{dt}\right|^{-2}, \quad (5)$$

as well as compression factor, k, which determine the ratio of dispersions of the particle coordinate and momentum,

$$k = \sigma_q/\sigma_p = |\varepsilon/(d\varepsilon/dt)|^2, \quad (6)$$

and the magnitudes of these dispersions

$$\sigma_q \geq \frac{\hbar}{2}\sqrt{\frac{k}{1-r^2}}, \quad \sigma_p \geq \frac{\hbar}{2}\sqrt{\frac{1}{k(1-r^2)}}, \quad (7)$$

can be determined using the complex normalized solution $\varepsilon(t) = e^{\varphi(t)}$, $\varphi(t) = \alpha(t) + i\beta(t)$ of the equation of motion of a classical oscillator with a varying frequency:

$$\frac{d^2\varepsilon}{dt^2} + \omega^2(t)\varepsilon = 0, \quad \varepsilon(0) = 1, \qquad \left.\frac{d\varepsilon}{dt}\right|_0 = i, \quad \omega(0) = 1. \quad (8)$$

In these relationships, $\omega(t)$ is the dimensionless frequency normalized to the characteristic frequency ω_0; t is the dimensionless time (normalized to ω_0^{-1}); $\varepsilon(t)$ is the dimensionless (normalized to $q_0 = \sqrt{\hbar/M\omega_0}$)

complex coordinate of the particle; and M is the reduced mass of the particle.

The substitution of $\varepsilon(t) = e^{\varphi(t)}$ leads to the following equations for real-valued functions:

$$\frac{d^2\alpha}{dt^2} + \left(\frac{d\alpha}{dt}\right)^2 - \exp(-4\alpha) = -\omega^2(t), \tag{9}$$

$$\beta(t) = \int_0^t \exp\{-2\alpha(t')\}dt', \tag{10}$$

which should use the initial conditions

$$\varphi(0) = \alpha(0) = \beta(0) = 0, \quad \left.\frac{d\varphi}{dt}\right|_0 = i, \quad \left.\frac{d\alpha}{dt}\right|_0 = 0, \quad \left.\frac{d\beta}{dt}\right|_0 = 1. \tag{11}$$

The final expression for the correlation coefficient can be derived from the formula

$$|r| = \left\{\left(\frac{d\alpha}{dt}\right)^2 \exp(4\alpha) \Big/ \left\{1 + \left(\frac{d\alpha}{dt}\right)^2 \exp(4\alpha)\right\}\right\}^{1/2}. \tag{12}$$

These relationships were generalized to the case in which the system under investigation (a nonstationary harmonic oscillator) is in the mixed state described by the density matrix [6, 15] or is in the state for which the action of a random force or the effect of random frequency fluctuations on CCS formation should be taken into account [10, 16, 17]. Analysis of specific mechanisms of CCS formation for various regimes of deformation of the potential well, as well as analysis of specific manifestation of this state in model and actual systems, was carried out for a monotonic asymptotic decrease or increase of the oscillator frequency [8, 9, 17], for a change of this frequency in a limited interval [13], for a periodic variation of this frequency [9, 11–13, 17], and in the presence of a random force and frequency fluctuation [10, 16, 17].

In [11, 12], apart from general regularities of CCS formation, the possibility of using these states for optimizing nuclear reactions at low energy in specific systems and for interpreting earlier experiments was considered.

General nontrivial features of a nuclear reaction induced by a particle in a correlated state with a strong energy fluctuation were considered in [16]. The most characteristic features are associated with the impossibility of the implementation of endoergic reactions with the prohibition of reactions with the formation of a long-lived intermediate state of the nucleus (i.e., with a sharp suppression of the channel leading to the formation of radioactive nuclei). These conclusions are in good agreement with the results of independent experiments (e.g., [1]).

It should be noted that CCS formation is a quite universal phenomenon that can occur not only in typical experimental conditions (a solid matrix or gas and almost stationary particles), but also for the motion of particles in the channeling regime in crystals with modulated parameters [21]. This can be used, for example, for optimizing "accelerator" fusion [22] with the participation of lattice nuclei that form the channel walls, as well as particles that were initially accelerated to a moderate energy or for generating coherent bremsstrahlung and Cherenkov radiation [23, 24].

An interesting aspect of optimization of the CCS method for increasing the potential barrier transparency was considered in [25], which was devoted to the nontrivial effect of temperature on the tunnel effect.

The possibility of the application of CCSs in biophysical processes was considered in [26, 27].

The analysis performed here shows the prospects and potentially high effectiveness of application of CCSs for nuclear fusion at high energies.

At the same time, it is obvious that all regimes of CCS formation considered earlier (except natural processes of formation of microcracks in metal hydrides during hydrogen loading [13] and "healing" of microinhomogeneities that emerge during the metabolism and growth of biological objects [26, 27]) presume the application of methods of action on the system that are difficult to implement in an experiment.

In particular, in an analysis of the method of controllable harmonic action, an idealized case of strictly monochromatic modulation of potential well parameters was used. Obviously, the actual action is always characterized by a finite spectral width. It should be recalled that the idea of CCS formation is based on the establishment of strictly definite phase relationships between different superposition eigenstates of the particle in the potential well, which necessitates analysis of the spectrum of this action. This circumstance becomes especially important due to the fact that CCS formation in some cases is characterized by a long duration ($\tau_c \gg \omega_0^{-1}, \Omega_0^{-1}$) [9]. Therefore, it is impossible to estimate a priori the effect of nonmonochromaticity on the CCS parameters (τ_c and the maximum attainable value of $|r|_{\max}$) and on the resultant effectiveness of CCS application for increasing transparency of the barrier for a specific nuclear interaction without performing the appropriate analysis.

Another clarification concerns the method of CCS formation during a monotonic (not oscillating) variation of the potential well parameters. Previously, two limiting scenarios of such a variation (unidirectional increase or analogous decrease in the frequency of a nonstationary oscillator) were considered, which in the case of an actual potential well corresponded, for example, to a monotonic decrease or increase in its width, and in the case of the action of an external magnetic field, to an increase or decrease in the amplitude of this field. In all these cases, the initial and final states of the system (oscillator size, microcrack size, L, and external magnetic-field strength, H) were substantially different. If we take the fact into account that

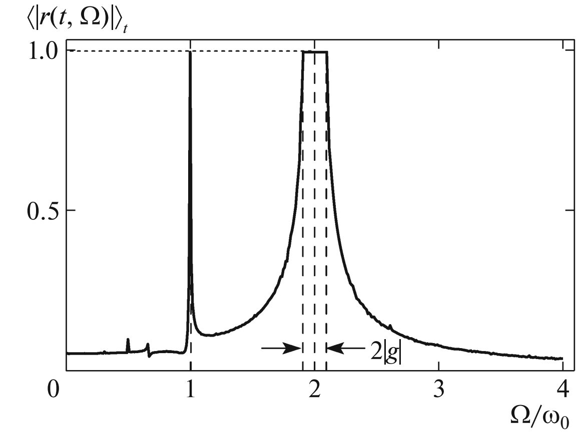

Fig. 1. The dependence of time-averaged correlation coefficient $\langle |r(t, \Omega)| \rangle_t$ on the ratio of frequency Ω of periodic modulation of potential well parameters to initial frequency ω_0 of particle oscillations in this well.

the formation of a CCS with a large correlation coefficient under such a monotonic perturbation is possible only when these states are considerably different [8, 9, 12, 13], this ultimately corresponded to nonoptimal experimental conditions (e.g., fracture of the system during cracking of metal hydrides) or necessitated the prolonged action of a very strong magnetic field prior to its monotonic variation or after its termination, etc. Obviously, in some cases (for example, under the action of an external magnetic field), a better scenario of CCS formation is that in which the external "reversible" action corresponds to a pulse that first rapidly transforms the system into an intermediate state and then returns it to the initial state. An analogous situation corresponds, for example, to the passage through the medium of a shock wave, such that the interatomic distance decreases at its leading edge (this actually corresponds to compression of the oscillator) and the system returns to the initial state at the trailing edge of the wave. Since the phase relationships in the quantum superposition state are different for different regimes of deformation of the potential well, the familiar results that were obtained for unidirectional modulation of the potential-well parameters cannot obviously be generalized to the case of alteration of processes with different directions and require separate analysis.

The questions that were mentioned above will be considered below using relationships (5)–(9) on the basis of specific laws of modulation of frequency $\omega(t)$.

3. THE FEATURES OF CCS FORMATION DUE TO A NONMONOCHROMATIC EXTERNAL ACTION

Let us consider the dynamics and limiting characteristics of CCS formation in two different versions of broadband modulation of the parameters of a nonstationary harmonic oscillator.

(a) Modulation due to the action with the structure that corresponds to a Gaussian-type function with a normalized integrated intensity, which is varied with the spectral density

$$F_a(\omega) = \frac{1}{\sqrt{\pi}\Delta\Omega}\exp\left\{-\left(\frac{\omega-\Omega}{\Delta\Omega}\right)^2\right\}, \tag{13a}$$

central frequency Ω, and frequency band $\Delta\Omega$. This function corresponds to the modulation of frequency of the nonstationary oscillator

$$\begin{aligned}\omega(t) &= \omega_0\{1 + g_a F_a(t)\} \\ &= \omega_0\{1 + g\sin(\Omega t)e^{-(\Delta\Omega t/2)^2}\},\end{aligned} \tag{14a}$$

$$\begin{aligned}g &= g_a/\sqrt{2\pi}, \\ F_a(t) &= (1/\sqrt{2\pi})\sin(\Omega t)e^{-(\Delta\Omega t/2)^2}.\end{aligned} \tag{15a}$$

(b) Modulation due to an action that is characterized by a uniform spectrum with central frequency Ω, frequency band $\Delta\Omega$, and the fixed spectral density

$$F_b(\omega) = 1/\Delta\Omega_0, \quad \Omega - \Delta\Omega \le \omega \le \Omega + \Delta\Omega/2. \tag{13b}$$

This function corresponds to the modulation of the oscillator frequency

$$\begin{aligned}\omega(t) &= \omega_0\{1 + g_b F_b(t)\} \\ &= \omega_0\left\{1 + g\left(\frac{\Delta\Omega}{\Omega}\right)\sin(\Omega t)\left(\frac{\sin(\Delta\Omega t/2)}{\Delta\Omega t/2}\right)\right\},\end{aligned} \tag{14b}$$

$$\begin{aligned}g &= \frac{g_b\Omega}{\Delta\Omega_0}, \\ F_b(t) &= \frac{1}{\sqrt{2\pi}}\left(\frac{\Delta\Omega}{\Delta\Omega_0}\right)\sin(\Omega t)\left(\frac{\sin(\Delta\Omega t/2)}{\Delta\Omega t/2}\right).\end{aligned} \tag{15b}$$

It was shown in our earlier publications [11, 17] that the maximum rate of CCS formation in the case of a monochromatic action on the oscillator (for $\Delta\Omega = 0$) corresponds to two frequencies, viz., direct resonance with frequency $\Omega = \omega_0$ and parametric resonance with a frequency close to $\Omega \approx 2\omega_0$ (Fig. 1). Therefore, it is most interesting to analyze the effect of nonmonochromaticity of the modulation process on CCS formation in the vicinity of these frequencies.

3.1. The Dynamics of CCS Formation under Nonmonochromatic Modulation of the Potential Well at Resonance Frequency $\Omega = \omega_0$ by a Function with Normalized Integrated Intensity

Let us consider CCS formation under the modulation of the potential-well parameters, which corresponds to spectral density (13a) and a time-dependent

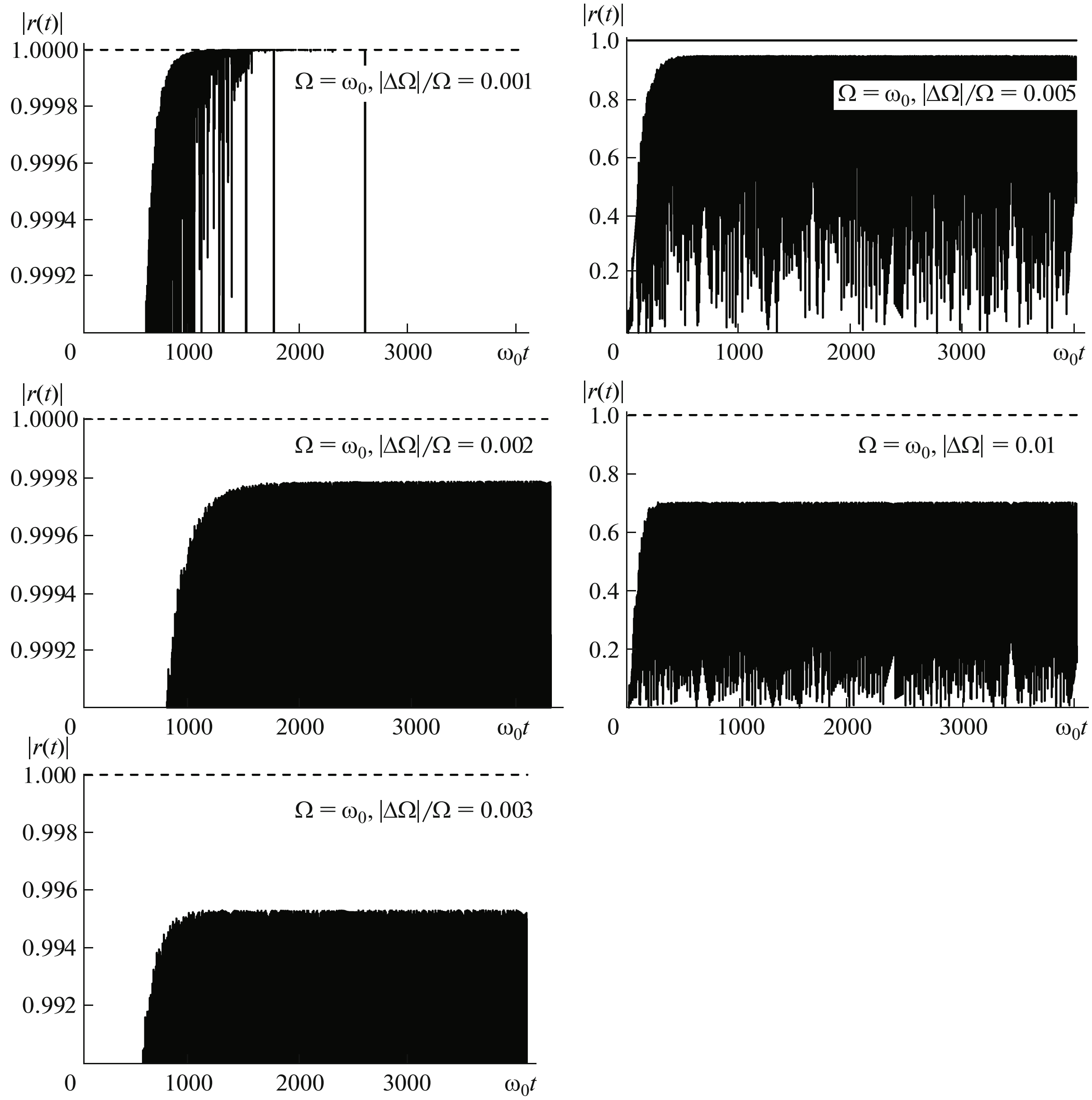

Fig. 2. The time dependence of the correlation coefficient for various monochromaticities of modulation of potential well parameters at resonance frequency $\Omega = \omega_0$. For values of $|\Delta\Omega|/\Omega \leqslant 0.003$ that correspond to $|r(t)|_{max} \to 1$, only the domain in which $|r(t)| \to 1$ is represented.

frequency of the classical oscillations of a particle in this well:

$$\omega(t) = \omega_0\{1 + g\sin(\omega_0 t)e^{-(\Delta\Omega t/2)^2}\} \equiv \omega_0\left\{1 + g\sin(\omega_0 t)\exp\left[-\left(\frac{\Delta\Omega}{2\Omega}\omega_0 t\right)^2\right]\right\}. \quad (16)$$

This modulation can be due, for example, to periodic variations of the well width

$$L(t) = L_0\{1 + g\sin(\omega_0 t)e^{-(\Delta\Omega t/2)^2}\}^{-1}, \quad L_0 = \sqrt{8V_{max}/M\omega_0^2} \quad (17)$$

or the well depth

$$V_{max}(t) = V_{max}(0)\{1 + g\sin(\omega_0 t)e^{-(\Delta\Omega t/2)^2}\}^2. \quad (18)$$

Analysis of CCS formation was carried out using relationships (9)–(12) and (14a) for a relatively small modulation index $|g| = 0.1$ analogously to earlier calculations for a monochromatic action [9, 11, 12].

Figure 2 shows the results of calculation of the dynamics of CCS formation under various types of nonmonochromatic modulation of the potential well ($|\Delta\Omega|/\Omega$ = 0.001, 0.002, 0.003, 0.005, 0.01) at frequency $\Omega = \omega_0$. These results show that the values of

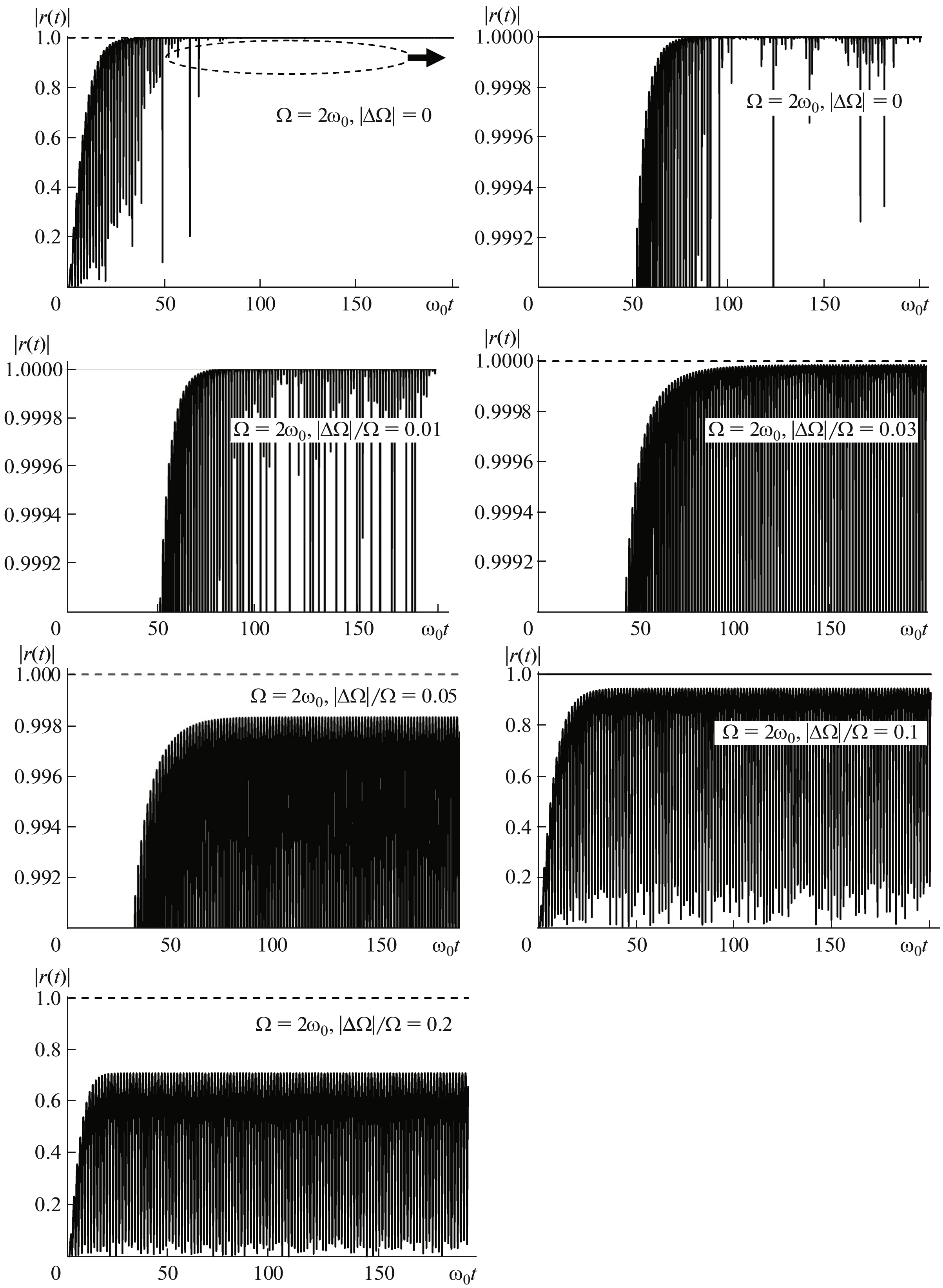

Fig. 3. The time dependence of the correlation coefficient for various spectral widths of modulation of the potential well parameters by Gaussian function (13a) with normalized integrated intensity in parametric resonance at frequency $\Omega = 2\omega_0$.

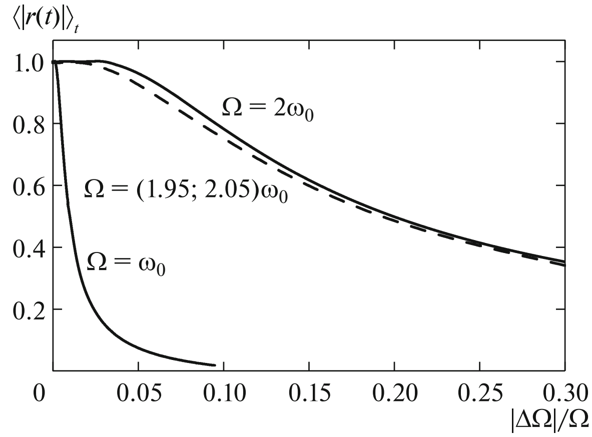

Fig. 4. Dependences of the time-averaged correlation coefficient $\langle|r(t, \Omega)|\rangle_t$ on the relative width of the spectrum of the potential well modulation by the Gaussian function (13a) with normalized integrated intensity for various modulation frequencies.

$|r|_{\max}$ and, hence $\langle|r(t)|\rangle_t$, rapidly decrease with increasing $|\Delta\Omega|$. In particular, the solution for $|r|_{\max}$ at $\Delta\Omega = 0$ and $t \geqslant 2 \times 10^3/\omega_0$ corresponds to a very large correlation effectiveness coefficient $G_{\max} \equiv 1/\sqrt{1 - r_{\max}^2} \geqslant 10^3$, while for $|\Delta\Omega|/\Omega = 0.002$ and the same value of time, we have $G_{\max} \approx 50$, and for $|\Delta\Omega|/\Omega = 0.01$, the decrease in this value is $G_{\max} \approx 2$. This means a very rapid decrease in quantities $|r|_{\max}$ and $G_{\max}$ is quite expected if we proceed from a very small half-width of a narrow resonance in the dependence of $\langle|r(t, \Omega|\rangle_t$ on frequency Ω shown in Fig. 1, as well as a sharp decrease in the spectral density of modulation (13a) upon an increase in the spectral width $|\Delta\Omega|$.

3.2. Peculiarities in CCS Formation under Nonmonochromatic Modulation of the Potential Well at the Parametric Resonance Frequency $\Omega = 2\omega_0$

3.2.1. The dynamics of CCS formation under frequency modulation by a function with normalized integrated intensity. Let us consider the effect of nonmonochromaticity of the modulation of the potential well parameters by a Gaussian function with a normalized integrated (and variable spectral) intensity on CCS formation in parametric resonance at $\Omega = 2\omega_0$, which corresponds to nonstationary oscillator frequency (13b)

$$\omega(t) = \omega_0\{1 + g\sin(2\omega_0 t)e^{-(\Delta\Omega t/2)^2}\}$$
$$\equiv \omega_0\left\{1 + g\sin(2\omega_0 t)\exp\left[-\left(\frac{\Delta\Omega}{\Omega}\omega_0 t\right)^2\right]\right\}, \quad (19)$$

which is characterized by the perturbation spectrum in the form of normalized Gaussian distribution (13a) with a peak at $\Omega = 2\omega_0$.

Figure 3 shows the results of calculation of the correlation coefficient for the following modulation parameters: $|\Delta\Omega|/\Omega = 0$, 0.01, 0.03, 0.05, 0.1, 0.2 for the same value of frequency modulation index $|g| = 0.1$. This results show that the efficiency of CCS formation with large values of $|r|_{\max}$ and $G_{\max}$ under modulation of the potential well parameters at parametric resonance frequency $\Omega = 2\omega_0$ remains very high even under nonmonochromatic modulation. In particular, the modulation with $|\Delta\Omega|/\Omega = 0.01$ makes it possible to form a CCS with $G_{\max} \equiv 1/\sqrt{1 - r_{\max}^2} \geqslant 10^3$ by instant $t \geqslant 10^2/\omega_0$, which is several orders of magnitude better than in the case of analogous modulation at frequency $\Omega = \omega_0$.

Figure 4 shows the results of calculation of the modulation efficiency at frequencies $\Omega = \omega_0$ and $\Omega = 2\omega_0$ (as well as at frequencies $\Omega = 1.95\omega_0$ and $\Omega = 2.05\omega_0$ for control) at $|g| = 0.1$ for CCS formation, based on the dependence of the time-averaged correlation coefficient

$$\langle|r(t)|\rangle_t = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2}|r(t)|dt \quad (20)$$

on the relative monochromaticity of potential well modulation. These results were obtained for averaging of the running value of $|r(t)|$ over the time interval $t_2 = 4000/\omega_0 \geqslant t \geqslant t_1 = 3950/\omega_0$.

These results visually demonstrate the advantages of using parametric modulation at frequency $\Omega = 2\omega_0$, which ensures the formation of highly effective CCS even under the broadband action with a low spectral density on the potential well.

3.2.2. Dynamics of CCS formation with frequency modulation of the parameters of a nonstationary oscillator by external action with a fixed spectral density and a variable frequency band. Let us consider another limiting case when a CCS is formed via the modulation of the potential well parameters due to the action that is characterized by a uniform spectrum with central frequency $\Omega = 2\omega_0$, with variable spectral width (frequency band) $\Delta\Omega$, and with fixed spectral density

$$F_b(\omega) = 1/\Delta\Omega_0,$$
$$2\omega_0 - \Delta\Omega/2 \leq \omega \leq 2\omega_0 + \Delta\Omega/2. \quad (21)$$

Here, $1/\Delta\Omega_0$ is the basic characteristic of the modulation spectrum. The dependence of the action that is characterized by such a spectrum was obtained above and is described by relationships (13b)–(15b). This function corresponds to the following form of the oscillator frequency modulation:

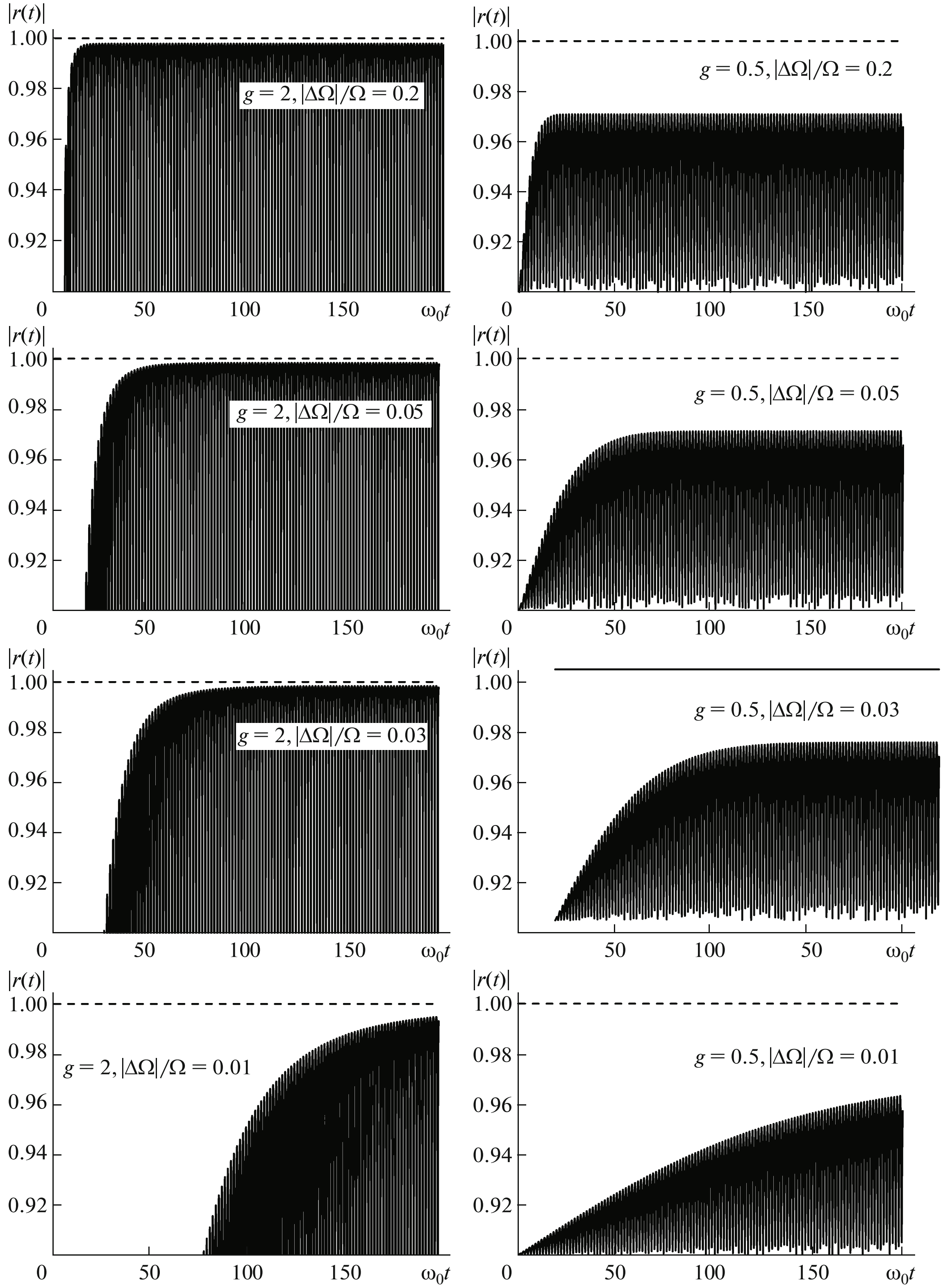

Fig. 5. The time dependence of the correlation coefficient for various band widths $\Delta\Omega$ in the spectrum of the external action with fixed spectral density $1/\Delta\omega_0$ for various modulation parameters g in the case of parametric resonance at frequency $\Omega = 2\omega_0$.

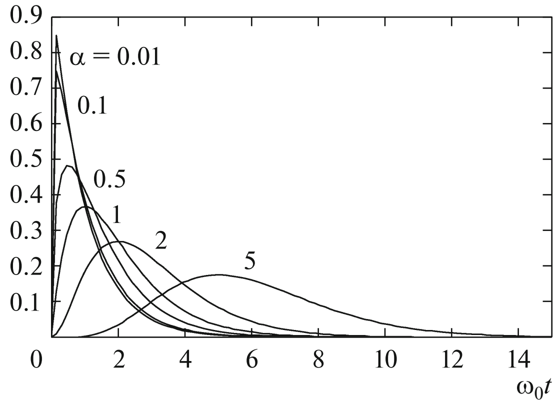

Fig. 6. The structure of the pulsed normalized frequency-modulation function of parameters of a nonstationary harmonic oscillator $F(t) = (\omega_0 t)^\alpha e^{-\omega_0 t}/\Gamma(\alpha + 1)$ for various values of the parameter α and leading edge duration $\Delta t = \alpha/\omega_0$.

$$\omega(t) = \omega_0\{1 + g_b F_b(t)\} = \omega_0\left\{1 + g\left(\frac{\Delta\Omega}{\Omega}\right) \times \sin(2\omega_0 t)\left(\frac{\sin[(\Delta\Omega/\Omega)\omega_0 t]}{(\Delta\Omega/\Omega)\omega_0 t}\right)\right\}, \quad g = \frac{g_b\Omega}{\Delta\Omega_0\sqrt{2\pi}}. \tag{22}$$

Figure 5 shows the results of numerical simulation of CCS formation using Eqs. (9)–(12) and frequency modulation (21), (22) of the nonstationary oscillator parameters by an external tunable action for various values of modulation parameter g in the case of parametric resonance at frequency $\Omega = 2\omega_0$. These results show that an increase in the modulation parameter g leads to a larger final value of $|r|_{\max}$. An increase in the spectral width $\Delta\Omega$ of the modulating action at a constant spectral density leads to a more rapid increase in the correlation coefficient to its limiting (depending on the given value of g) value $|r|_{\max}$, which is approximately equal to 0.998 for $g = 2$ and 0.972 for $g = 0.5$.

The former conclusion appears to be quite expected and obvious, while the latter conclusion is paradoxical and at first glance contradicts the results that were obtained earlier in Section 3.2.1 and presented in Fig. 3. According to these results, an increase in spectral width $|\Delta\Omega|$ normalized in the intensity of modulating action leads to a sharp decrease in the final value of $|r|_{\max}$. However, this apparent contradiction can easily be explained by the fact that in the case of a modulating action with a constant spectral intensity, an increase in $|\Delta\Omega|$ leads to an increase in the combined frequency modulation parameter $g|\Delta\Omega|/\Omega \equiv g_b|\Delta\Omega|/\Delta\Omega_0\sqrt{2\pi}$ in relationship (22), while modulation in the form of normalized Gaussian distribution (13a) with the center at $\Omega = 2\omega_0$ and with a variable $|\Delta\Omega|$ does not affect this amplitude. Accordingly, the spectral intensity in the case of normalized action with increasing $|\Delta\Omega|$ decreases, while with non-normalized action it remains constant (21) and equal to $F_b(\omega) = 1/\Delta\Omega_0$. If we take the fact into account that the rate of increase in $|r|_{\max}$ and $\langle|r(t)|\rangle_t$ increases very sharply with an increasing frequency modulation parameter, this fully explains the apparent contradiction.

4. CCS FORMATION UNDER A PULSED IRREVERSIBLE MODULATING ACTION ON THE POTENTIAL WELL

In this section, we consider the peculiarities of CCS formation under a reversible variation of parameters of a nonstationary harmonic oscillator in which the particle that is under investigation is located; this variation corresponds to pulsed modulation of the oscillator frequency $\omega(t)$ with various durations and different structures of the leading and trailing edges. It is convenient to analyze such a process using the expressions

$$\omega(t) = \omega_0\{1 + gF(t)\}, \quad F(t) = (\omega_0 t)^\alpha e^{-\omega_0 t}/\Gamma(\alpha+1). \tag{23}$$

Here, $F(t)$ is the normalized frequency modulation function and $\Gamma(\alpha + 1)$ is the gamma function. The expression for $F(t)$ shows that the duration of leading edge of the frequency modulation is $\Delta t = \alpha/\omega_0$. The explicit form of function $F(t)$ is shown in Fig. 6 for various values of parameter α. Small values of parameter $\alpha \ll 1$ correspond to a sharp decrease in the duration of the leading edge of function $F(t)$ and an increase in the amplitude. The condition $\alpha > 1$ corresponds to a nearly symmetric pulse with smoothly varying edges.

The dynamics of CCS formation (of variation of correlation coefficient $|r(t)|$ of the particle) in a parabolic potential well were calculated using Eqs. (9)–(12) and frequency modulation function (23). Figure 7 show the results of calculation of $|r(t)|$ for different values of modulation index $g = 100$, 50, 10 and various leading edge durations $\Delta t = \alpha/\omega_0$ of reversible pulsed action (23).

It can be seen that the rate of variation of $|r(t)|$ and limiting values $|r(t)|_{\max}$ depend on g as well as Δt so that the largest values of $|r(t)|_{\max}$ correspond to a large value of g and small Δt. These results also show that for a long leading edge duration, the resultant values of $|r(t)|_{\max}$ are small, even for a large value of g. This follows directly, in particular, from comparison of the curves in the lower row in Fig. 7. These curves show that for a large value of $\alpha = \omega_0\Delta t = 5$, the maximum correlation coefficient and correlation effectiveness coefficient change insignificantly and remain small ($|r(t)|_{\max} \approx 0.3$–$0.5$ and $G_{\max} \equiv 1/\sqrt{1 - r_{\max}^2} \approx 1.05$–$1.15$) even for a considerable variation of $g = 10$–100. It should be noted for comparison that for a small leading edge duration (for $\alpha = \omega_0\Delta t = 0.01$) and the same value of $g = 100$, these coefficients increase to values

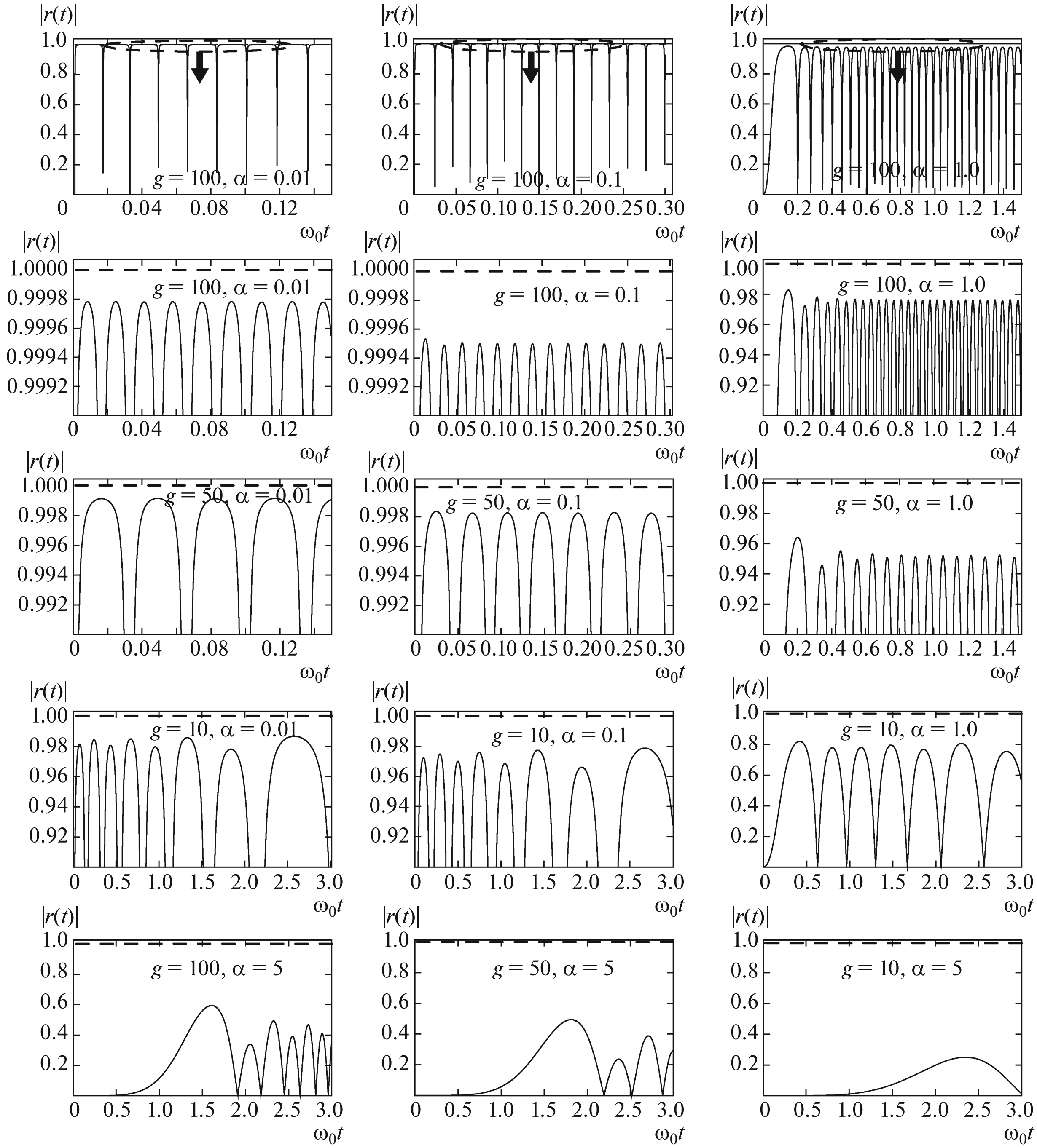

Fig. 7. The general form (upper line) and detailed structure of the time dependence of correlation coefficient $|r(t)|$ for various modulation indices g and various leading edge durations $\alpha = \omega_0 \Delta t$ of a reversible pulsed action $F(t) = (\omega_0 t)^{\alpha} e^{-\omega_0 t}/\Gamma(\alpha + 1)$ on the potential well parameters.

$|r(t)|_{\max} \approx 0.9995–0.9998$ and $G_{\max} \equiv 1/\sqrt{1 - r_{\max}^2} \approx 32–50$, which is sufficient for increasing the transparency of potential barrier by a factor of $D_{|r| = 0.9995–0.9998}/D_{|r| = 0} \approx 10^{50}–10^{100}$.

Our calculations indicate that effective CCS formation process can occur only in the case of a short edge of the pulse that simulates the oscillator frequency provided that the duration of this edge is much smaller than the period of the initial oscillator frequency ω_0. This conclusion fully confirms the results that were obtained earlier [9, 11, 12, 17] for a rapid monotonic "switching on" or "switching off" of the perturbation of frequency $\omega(t)$, which corresponds to the short leading or trailing edge for a constant (in fact, infinitely long) trailing or leading edge, respectively.

It should be noted that since the duration of the trailing edge with the pulse-modulation frequency changes insignificantly upon a strong change in the duration of the leading edge $\Delta t = \alpha/\omega_0$ (see Fig. 6), the effect of the trailing edge characteristics of the pulsed frequency modulation on CCS formation and on the change in $|r(t)|$ upon such a pulsed action remains unclear. To clarify this effect, we analyzed the CCS formation process with a pulsed variation of frequency

$$\omega(t) = \omega_0\{1 + gF(t)\},$$
$$F(t) = (1/\sqrt{\pi}\Delta t)e^{-[(t-t_0)/\Delta t]^2} \quad (24)$$

using the symmetric function $F(t)$ with variable and identical leading and trailing edges. The results of the calculations show that CCS formation in this case is an extremely ineffective process and $|r(t)|_{max} \approx 0$ for any value of Δt. This result is due to the fact that upon symmetric variation of resultant function $F(t)$, phase variations of the superposition states of the particle, which are formed on the leading edge, are compensated completely on the trailing edge.

5. CONCLUSIONS

Our results indicate that CCS formation can be effective when nonmonochromatic (or even in the form of a wide frequency band) modulation of the parameters of a nonstationary harmonic oscillator is used provided that the spectral density of this modulation is high. A better case corresponds to narrow-band quasi-monochromatic modulation of frequency of this oscillator. The amplitude of this modulation, which should be large enough, plays the leading role in this case. For monochromatic modulation, a change in the parameters of the oscillator at parametric resonance frequency $\Omega \approx 2\omega_0$ (or in the limits of the band near this resonance) is found to be much more effective than for analogous modulation in the frequency range in the vicinity of "direct" resonance at frequency $\Omega = \omega_0$.

These results substantially supplement the recommendations that were obtained earlier [9, 11, 12, 17] using monochromatic resonant pumping for frequency modulation of the oscillator that contains the given particle. These recommendations also considerably supplement the results that were described in [10] for a quite idealized delta-correlated fluctuation of parameters that determine the characteristics of frequency modulation.

The second (of those considered above) method of CCS formation by short-term pulsed modulation of the oscillator frequency (23) has not been investigated earlier. The results that were obtained above indicate that the best result with such a modulation corresponds to the action of asymmetric pulses with a short leading edge and a long trailing edge (or vice versa). This method can be used for optimizing a nuclear interaction at low particle energy in a simple experimental setup. One of the easiest ways to obtain such a modulation is associated with cyclotron resonance in a varying magnetic field.

It is well known that the Schrödinger equation for a charged particle of mass M and charge q in a magnetic field of strength H is analogous to the equation for a harmonic oscillator with the same wavefunctions, equidistant energy spectrum $E_n = n\hbar\omega$, $n = 1, 2, \ldots$, and frequency $\omega = qH/Mc$. In this case, the application of the variable magnetic field

$$H(t) = H_0\{1 + g(\omega_0 t)^{\alpha}e^{-\omega_0 t}/\Gamma(\alpha+1)\} \quad (25)$$

corresponds to the case with a nonstationary harmonic oscillator with frequency (23)

$$\omega(t) = \omega_0\{1 + g(\omega_0 t)^{\alpha}e^{-\omega_0 t}/\Gamma(\alpha+1)\}.$$

Let us estimate the conditions in which the application of such a pulsed magnetic field leads to the formation of an effective CCS. The results that are presented in Fig. 7 show that for a rapid formation of a CCS with $|r(t)|_{max} \approx 0.9998$ and $G_{max} \approx 50$, it is necessary that the leading edge duration of the magnetic-field pulse be shorter than $\Delta t = \alpha/\omega_0 = 0.01/\omega_0$, and the amplitude value of this field be

$$H_{max}(\Delta t)$$
$$= H_0\{1 + g(\omega_0\Delta t)^{\alpha}e^{-\omega_0\Delta t}/\Gamma(\alpha+1)\} \approx 86H_0. \quad (26)$$

If we proceed from the quite realistic assumption that a pulsed magnetic field with amplitude $H_{max}(\Delta t) \approx$ 10 kOe can easily be obtained in an experiment, we find that $\omega_0 \approx 5 \times 10^4$ Hz and $\Delta t \approx 2 \times 10^{-7}$ s for a gas (plasma) of particles with mass M_d and charge $q = e$ like that for a deuteron. The duration of the trailing edge in this case is $\tau \approx 5/\omega_0 \approx 10^{-4}$ s. Such improved (with a smaller value of Δt and a higher value of $H_{max}(\Delta t)$) magnetic-field pulses can easily be obtained using the corresponding magnetic-field pulses (see, for example, [28]).

For such parameters, the transparency of the tunnel barrier (e.g., for the dd fusion reaction) increases from the value of $D_{r=0} \approx 10^{-80}$, which is "conventional" for low temperatures, to $D_{|r|_{max}=0.9998} \approx 0.1$.

For heavier ions with mass $M = kM_d$, we have $\omega_0 \approx$ $(5 \times 10^4/k)$ Hz and $\Delta t \approx 2 \times 10^{-7}k$ s. Upon an increase in $H_{max}(\Delta t)$ and the corresponding decrease in $\Delta t =$ α/ω_0, the values of $|r|_{max}$ and G_{max} rapidly increase, which paves the way for implementation of highly effective nuclear reactions with the participation of heavy nuclei.

It is interesting to analyze the effect of the number density and temperature of the gas (plasma) in which such a process takes place on the "pulsed" method of CCS formation that was considered above. Obviously, the collision of the charged particle under investigation with other atoms and ions corresponds to a random force that acts on a nonstationary oscillator. This problem was considered earlier on the basis of relevant stochastic equations for various types of frequency modulation of an oscillator (high-frequency resonant modulation with $\Omega = \omega_0$, $2\omega_0$ [9], low-frequency nonresonant modulation with $\Omega \ll \omega_0$ [16], and monotonic modulation in the case of the formation of microcracks in metal hydrides [13]). As applied to the "pulsed" method that is considered here, we can obtain simple estimates based on the natural assumptions according to which phase relationships for CCS formation are not violated if the mean time $1/\sigma n v$ between two collisions of particles in the gas (plasma) is much longer than the time of CCS formation. It follows from Fig. 7 that in the optimal case (from those considered here) with parameters $g = 100$ and $\alpha = 0.01$, this time is $\tau_c \approx \alpha/\omega_0 = \Delta t$. We assume for estimates that $v \approx \sqrt{kT/M}$ is the mean velocity of particles in the gas and $\sigma \approx 3 \times 10^{-16}$ cm^2 is the total elastic scattering cross section at a low energy. Ultimately, we find that the above scenario of CCS formation can be realized if the number density of deuterium atoms (ions) for $H_{max}(\Delta t) \approx 10$ kOe and temperature $T = 300$ K is lower than $n_{cr} \approx 10^{17}$ cm^{-3}. Upon an increase in $H_{max}(\Delta t)$, the value of n_{cr} increases, while in a gas (plasma) of heavier atoms, it decreases.

Comparison of the results for n_{cr} with the data that were obtained in [16], where CCS formation via low-frequency nonresonant frequency modulation at the frequency $\Omega = 10^{-4}\omega_0$ was considered for an analogous gas (plasma) with allowance for collisions, shows that when such a pulsed frequency modulation is used, the critical number density, n_{cr}, of the gas is several orders of magnitude higher, which substantially optimizes the experimental conditions and does not require a high vacuum. This difference can easily be explained taking the fact into account that for low-frequency modulation [16], the minimum time of CCS formation ($\tau_c \approx 0.01/\Omega = 10^2/\omega_0$ is 10^4 times longer than $\tau_c \approx \alpha/\omega_0 = \Delta t$, which, accordingly, leads to much more stringent requirements on the admissible critical density, n_{cr}, as compared to those in [16].

The results that characterize CCS formation for a very significant increase in the potential barrier transparency and, hence, for "giant" acceleration of nuclear reactions at a low energy (temperature) can be used to explain the results of experiments in which anomalous nuclear processes were observed.

In particular, our results can be used for clarifying the mechanism of fundamental nuclear and isotope transformations [28–31] during the collapse of a small target as a result of the action of a high-current pulsed electron beam that generates strong magnetic-field pulses with a duration of approximately 10^{-8} s and a much shorter leading edge duration, Δt.

These results are applicable to the data that were obtained in [32], where it was shown that in switches in which commutation of high currents (1–50 kA) is performed in high-current industrial mains under a high voltage (up to 5 kV), a substantial change in the isotope composition of structural material from which these switches are made is observed after long service period. In particular, the scale of isotope transformations in Fe and Ti was 3–5%. It is important that in all tested devices in which isotope anomalies were detected, an electric (plasma) arc appeared at the instant of the interruption of the high current; self-compression for such an arc in the pinch effect inevitably leads to the generation of a high-intensity magnetic-field pulse.

In experiments [33], significant isotope changes (at a level of 5–7%) were detected during the explosion of wires and foils immersed in a liquid, through which submillisecond high-current pulses with a total energy of 20–30 kJ were passed. Nuclear transformations with analogous effectiveness were also observed in experiments [34] in which a considerable decrease in the Zn concentration was detected upon the passage of high-power submicrosecond current pulses through an aqueous solution of $ZnSO_4$ salt.

The characteristic change in the electric current in these experiments led to the generation of magnetic-field pulses with structures and parameters that approximately correspond to the above requirements for CCS formation and for a radical increase in tunneling probability.

REFERENCES

1. G. Levi, E. Foschi, B. Höistad, R. Pettersson, L. Tegnér, and H. Essén, *Lugano Report, October 6, 2014. Observation of Abundant Heat Production from a Reactor Device and of Isotopic Changes in the Fuel.* http://www.sifferkoll.se/sifferkoll/wp-content/uploads/2014/10/LuganoReport-Submit.pdf.
2. E. Schrodinger, Ber. Kgl. Akad. Wiss., Berlin 24, 296 (1930).
3. H. P. Robertson, Phys. Rev. A: At., Mol., Opt. Phys. **35**, 667 (1930).
4. V. Dodonov, E. Kurmishev, and V. Man'ko, Phys. Lett. A **79**, 150 (1980).
5. V. V. Dodonov and V. I. Man'ko, Tr. Fiz. Inst. im. P. N. Lebedeva, Akad. Nauk SSSR **183**, 71 (1987).
6. V. V. Dodonov, A. V. Klimov, and V. I. Man'ko, Tr. Fiz. Inst. im. P. N. Lebedeva, Akad. Nauk SSSR **200**, 56 (1991).
7. V. V. Dodonov, A. B. Klimov, and V. I. Man'ko, Phys. Lett. A **220**, 41 (1996).
8. V. I. Vysotskii and S. V. Adamenko, Tech. Phys. **55** (1), 613 (2010).
9. V. I. Vysotskii, M. V. Vysotskyy, and S. V. Adamenko, J. Exp. Theor. Phys. **114** (2), 243 (2012).
10. V. I. Vysotskii, S. V. Adamenko, and M. V. Vysotskyy, J. Exp. Theor. Phys. **115** (4), 551 (2012).

11. V. I. Vysotskii and M. V. Vysotskyy, Eur. Phys. J. A **49**, 99 (2013).
12. V. I. Vysotskii, S. V. Adamenko, and M. V. Vysotskyy, Ann. Nucl. Energy **62**, 618 (2013).
13. V. I. Vysotskii and M. V. Vysotskyy, J. Exp. Theor. Phys. **118** (4), 534 (2014).
14. V. V. Dodonov and A. V. Dodonov, J. Russ. Laser Res. **35**, 39 (2014).
15. A. V. Dodonov and V. V. Dodonov, Phys. Lett. A **35**, 1071 (2014).
16. V. I. Vysotskii and M. V. Vysotskii, J. Exp. Theor. Phys. **120** (2), 246 (2015).
17. V. I. Vysotskii and M. V. Vysotskyy, Curr. Sci. **108**, 524 (2015).
18. L. Maccone and P. K. Arun, Phys. Rev. Lett. **113**, 260401 (2014).
19. C. Anastopoulos and J. J. Halliwell, Phys. Rev. D: Part. Fields **51**, 6870 (1995).
20. J. Halliwell and A. Zoupas, Phys. Rev. D: Part. Fields **52**, 7294 (1995).
21. V. I. Vysotskii, S. V. Adamenko, and M. V. Vysotskii, J. Surf. Invest. **6** (2), 369 (2012).
22. V. I. Vysotskii, M. V. Vysotskyy, and S. Bartalucci, Ann. Nucl. Energy **62**, 613 (2013).
23. V. A. Bazylev and N. K. Zhevago, Radiation of Fast Charged Particles in Matter and External Fields (Nauka, Moscow, 1987) [in Russian].
24. M. V. Vysotskyy and V. I. Vysotskii, Nucl. Instrum. Methods Phys. Res., Sect. B **252**, 75 (2006).
25. V. N. Chernega, J. Russ. Laser Res. **34**, 168 (2013).
26. V. I. Vysotskii and A. A. Kornilova, Ann. Nucl. Energy **62**, 626 (2013).
27. V. I. Vysotskii and A. A. Kornilova, Curr. Sci. **108**, 636 (2015).
28. *Controlled Nucleosynthesis: Breakthroughs in Experiment and Theory*, Ed. by S. Adamenko, F. Selleri, and Alwyn van der Merwe (Springer-Verlag, Berlin, 2007).
29. S. V. Adamenko and V. I. Vysotskii, Found. Phys. **34**, 1801 (2004).
30. S. V. Adamenko and V. I. Vysotskii, Found. Phys. Lett. **17**, 203 (2004).
31. S. V. Adamenko and V. I. Vysotskii, Found. Phys. Lett. **19**, 21 (2006).
32. A. S. Agapov, V. A. Kalenskii, Ch. B. Kaitusov, A. V. Malyshev, G. V. Ryabov, A. V. Steblevskii, L. I. Urutskoev, and D. V. Filippov, in *Proceedings of the 13th Russian LENTChE Conference, Dagomys, Sochi, Russia, 2004.*
33. L. I. Urutskoev, V. I. Liksonov, and V. G. Tsinoev, Ann. Found. Louis Broglie **27**, 701 (2002).
34. V. V. Krymskii and V. F. Balakirev, Dokl. Phys. Chem. 385 (4–6), 197 (2002).

Translated by N. Wadhwa