A Possible Scenario for the Onset of Cold Fusion in Deuterated Metals

R. H. Parmenter
Department of Physics
University of Arizona
Tucson, Arizona 85721

July 24, 1994

Abstract

It is suggested that a pair of deuterons in a deuterated metal may resonant-tunnel through the Coulomb barrier separating them and form a helium isomer characterized by \( L = 1, S = 1 \), and odd parity. The isomer can decay in several ways, each of which is analyzed, resulting in simple explanations of several surprising experimental observations: the very large tritium/neutron ratios, the association of excess heat with helium production, the difficulty in obtaining reproducible results. The validity of the scenario can be checked by looking for the nuclear magnetic moment of the helium isomer.

In a series of three papers, Parmenter and Lamb [1] demonstrated that conventional physics could explain the level of fusion processes observed by Jones et al. [2] in deuterated metals. A pair of deuterons can be trapped in a potential well in a tetrahedral cavity in palladium metal. The deuterons continually bang into the Coulomb barrier separating them with the characteristic frequency \( \omega_0 \) of the harmonic-oscillator well. A large number of conduction electrons contribute to the screening of the Coulomb field of each deuteron. This screening is far more effective than is the corresponding screening by a pair of electrons in an isolated deuterium molecule. Thus the metal provides an environment conducive to enhanced Coulomb barrier penetration and resultant nuclear interaction.

The apparent lack of reproducibility of some experimental results has led some to insist that everything associated with cold fusion is spurious, with the possibility of some scientific fraud [3]. Nevertheless, there is growing evidence [4] that under certain conditions the amount of heat generated is well beyond what was observed by Jones et al. [2]. For a recent review of the field, see V.A. Chechin et al. [5].

Turner [6] originally suggested the possibility of a mobile deuteron resonant tunneling through the Coulomb barrier associated with a periodic array
of deuterons in interstitial positions in the metal. This model has been further pursued by Bush [7] and by Bass [8], simplifying the model to a one-dimensional array. Kim [9] suggested the possibility of resonant tunneling associated with the Coulomb barrier separating two deuterons, but he apparently did not pursue the matter. The purpose of the present paper is to examine the consequences of the possibility of resonant tunneling of the Coulomb barrier separating two deuterons. I will show that this idea leads to simple explanations of several surprising experimental observations. Furthermore, the model leads to an unambiguous prediction that should not be too difficult to check or refute experimentally.

In order for resonant tunneling of the Coulomb barrier by a pair of deuterons to occur, it is necessary that there be some excited state of the pair having an energy equal or close to zero when the separation distance is in the nuclear interaction range $R_0$. I assume $R_0$ is the experimentally measured rms diameter of the $\alpha$ particle, i.e. $R_0 = 3.22 \times 10^{-13} \text{ cm}$. After all, the ground state of the deuteron pair is the $\alpha$ particle. The ground-state binding energy is

$$E_0 = (2m_D - m_\alpha)c^2 = 23.8458 \text{MeV.} \quad (1)$$

I take the simplest possible model for the nuclear interaction potential as a function of the separation distance of the pair, namely a spherically symmetric rectangular potential well of radius $R = R_0 \eta$, where $\eta$ is some number close to one. Since each deuteron has a finite size, one might expect $\eta$ to be slightly less than one. Since the height of the Coulomb barrier is approximately

$$E_C = (e^2/R_0) = 0.447 \text{MeV}, \quad (2)$$

we may consider the potential well to have an infinitely high wall when considering a zero-energy state. Thus the energies of the excited states are

$$E_{nl} = (\hbar^2/m_D R^2)(X_{nl}^2 - X_{10}^2) - E_0, \quad (3)$$

where

$$j_l(X_{nl}) = 0, \quad n = 1, 2, \ldots, \quad l = 0, 1, 2, \ldots, \quad \{ \begin{array}{c} (\hbar^2/m_D R^2) = 2.002176 \text{MeV.} \end{array} \} \quad (4)$$

(Here $j_l$ is the spherical Bessel function of order $l$.) Now the only reasonable possibility of a zero-energy excited state is the first excited state $E_{11} = 0$. Since $X_{10} = \pi, X_{11} = 4.4934095$, we get a vanishing $E_{11}$ when

$$\eta = 0.93091, \quad R = 2.9975 \times 10^{-13} \text{ cm.} \quad (5)$$

This is a zero-energy $p$-wave resonance of the two deuterons. The fact that it is $p$-wave ($L = 1$) is crucial to everything that follows.

If the spatial portion of the $D_2$ wavefunction is in a $p$-state, then this portion of the wavefunction is antisymmetric under interchange of the spatial coordinates of the two particles. Thus the spin portion of the wavefunction must be
antisymmetric under interchange of spin coordinates (para-$D_2$), since deuterons are bosons. Each deuteron having a spin of 1, the pair might have spin 2, 1, or 0, but total spin 1 is the only choice that is antisymmetric.

Let $r_1$ and $r_3$ be the positions of the two protons of the deuteron pair, and let $r_2$ and $r_4$ be the positions of the two neutrons. The center-of-mass position of the deuteron pair is

$$R_c = \frac{1}{4}(r_1 + r_2 + r_3 + r_4).$$

(Here I have made the approximation that neutrons and protons have the same mass.) Take

$$R_{ij} = \frac{1}{2}(r_i + r_j), \quad r_{ij} = (r_i - r_j).$$

Thus $R_{12}, R_{12}$ are the center-of-mass and internal positions, respectively, of one deuteron, while $R_{34}, R_{34}$ are the corresponding quantities for the other deuteron. The $D_2$ wavefunction can be written

$$\psi(R_c, r_{12}, r_{34}, R_{12} - R_{34}, \sigma_{12}, \sigma_{34}) = \varphi(R_c)\psi_0(r_{12})\psi_0(r_{34})\psi_1(R_{12} - R_{34})\Phi(\sigma_{12}, \sigma_{34}).$$

$\varphi(R_c)$ is the center of mass wavefunction; $\psi_0(r_{12})$ and $\psi_0(r_{34})$ are the internal wavefunctions of the two deuterons; $\psi_1(R_{12} - R_{34})$ is the $p$ wavefunction describing the motion of one deuteron relative to the other; $\sigma_{12}$ and $\sigma_{34}$ are the spin coordinates of the two deuterons. I take the total spin state to have $S = 1, S_z = 0$, so that

$$\Phi(\sigma_{12}, \sigma_{34}) = |1, 0\rangle_{12,34} = \frac{1}{\sqrt{2}}[|1, 1\rangle_{12}|1, 1\rangle_{34} - |1, -1\rangle_{12}|1, -1\rangle_{34}]$$

When the two deuterons are within the nuclear interaction range, one should more properly consider a 4-fermion properly antisymmetrized wavefunction. Such a wavefunction is

$$\Psi_1 \equiv \frac{1}{\sqrt{2}}[\psi(R_c, r_{12}, r_{34}, R_{12} - R_{34}, \sigma_{12}, \sigma_{34}) - \psi(R_c, r_{14}, r_{23}, R_{14} - R_{23}, \sigma_{14}, \sigma_{23})].$$

This wavefunction is properly antisymmetric under interchange of proton coordinates and under interchange of neutron coordinates. We have a zero-energy $^2\text{He}^4$ nuclear isomer (metastable excited state) of negative parity, total $L = 1$, and total $S = 1$, so that the total $J$ may be 2, 1, or 0. We designate this isomer by the symbol $^2\text{He}^{4\ast}$. This excited state would not be noticed experimentally by low-energy deuteron-deuteron scattering or by photodisintegration of the $\alpha$ particle.
Let us consider four channels by which the isomer can decay. They are, respectively,

\[ \begin{align*}
1) & \quad ^2\text{He}^{4*} \rightarrow ^2\text{He}^4 + \gamma M_1 + \gamma E_1, \\
2) & \quad ^2\text{He}^{4*} \rightarrow ^2\text{He}^4 + \gamma M_1, \\
3) & \quad ^2\text{He}^{4*} \rightarrow ^1\text{H}^3 + \gamma M_1 + p, \\
4) & \quad ^2\text{He}^{4*} \rightarrow ^2\text{He}^3 + \gamma M_1 + n.
\end{align*} \] (11)

Here $\gamma E_1$ is an electric-dipole photon, $\gamma M_1$ a magnetic-dipole photon; $n$ is a neutron, $p = ^1\text{H}^1$ is a proton, $t = ^1\text{H}^3$ is a triton, and $\alpha = ^2\text{He}^4$ is an $\alpha$ particle. The values of total energy release for each channel are

\[ \begin{align*}
Q_1 &= 23.846 \text{ MeV}, \\
Q_2 &= 23.846 \text{ MeV}, \\
Q_3 &= 4.033 \text{ MeV}, \\
Q_4 &= 3.269 \text{ MeV}.
\end{align*} \] (12)

Notice that in every channel a magnetic-dipole photon is emitted. This has the effect of converting an $S = 1$ state into an $S = 0$ state, Thus $|1, 0\rangle_{12, 34}$ is converted into

\[ |0, 0\rangle_{12, 34} = \frac{1}{\sqrt{6}} \left[ (|1, -1\rangle_{12}|1, 1\rangle_{34} - 2|1, 0\rangle_{12}|1, 0\rangle_{34} + |1, 1\rangle_{12}|1, -1\rangle_{34} \right]. \] (13)

A portion of this state represents the situation where the two protons have antiparallel spins and the two neutrons have antiparallel spins. To see this, write $|0, 0\rangle_{12, 34}$ in terms of the spin orbitals for the four fermions,

\[ |0, 0\rangle_{12, 34} = \frac{1}{\sqrt{6}} \left[ (|1\rangle_1|1\rangle_2|1\rangle_3|1\rangle_4 + |1\rangle_1|1\rangle_2|1\rangle_3|1\rangle_4 \right. \\
- (|1\rangle_1|1\rangle_2 + |1\rangle_1|1\rangle_2)(|1\rangle_3|1\rangle_4 + |1\rangle_3|1\rangle_4) \\
= \frac{1}{\sqrt{6}} \left[ (|1\rangle_1|1\rangle_3 - |1\rangle_1|1\rangle_3)(|1\rangle_2|1\rangle_4 - |1\rangle_2|1\rangle_4 \right. \\
- |1\rangle_1|1\rangle_3|1\rangle_2|1\rangle_4 - |1\rangle_1|1\rangle_3|1\rangle_2|1\rangle_4 \right] \\
= \frac{1}{\sqrt{6}} \left[ 2|0, 0\rangle_{13}|0, 0\rangle_{24} - |1, 1\rangle_{13}|1, -1\rangle_{24} - |1, -1\rangle_{13}|1, 1\rangle_{24} \right]. \] (14)

The first term of the final result represents the situation where the two protons have antiparallel spins and the two neutrons also have antiparallel spins; the second term corresponds to both protons having spin up, both neutrons spin down; the third term corresponds to both protons having spin down, both neutrons spin up. Similarly, we have

\[ |0, 0\rangle_{14, 23} = \frac{1}{\sqrt{6}} \left[ -2|0, 0\rangle_{13}|0, 0\rangle_{24} - |1, 1\rangle_{13}|1, -1\rangle_{24} - |1, -1\rangle_{13}|1, 1\rangle_{24} \right]. \] (15)
In terms of the four decay channels, the only portions of both Eqs (14) and (15) that are of interest are the portions corresponding to antiparallel spins of protons and of neutrons. Thus, with probability \((2/3)\), a magnetic-dipole emission leads to a state

\[
\Psi_2 = |0, 0\rangle_13 |0, 0\rangle_24 \varphi(R_c) \times \frac{1}{\sqrt{2}} \left[ \psi_0(r_{12})\psi_0(r_{34})\psi_1(R_{12} - R_{34}) + \psi_0(r_{14})\psi_0(r_{23})\psi_1(R_{14} - R_{23}) \right]. \tag{16}
\]

Consider the three states described by

\[
\Psi_3 = |0, 0\rangle_13 |0, 0\rangle_24 \varphi(R_c) \psi'_0(r_{13})\psi'_0(r_{24})\psi'_1(R_{13} - R_{24}), \tag{17}
\]

\[
\Psi_4 = |0, 0\rangle_13 |0, 0\rangle_24 \varphi(R_c) \times \frac{1}{\sqrt{2}} \left[ \psi_2(r_{24}, r_{1} - R_{24})\psi_3(R_{124} - r_{3}) + \psi_2(r_{24}, r_{3} - R_{24})\psi_3(R_{234} - r_{1}) \right], \tag{18}
\]

\[
\Psi_5 = |0, 0\rangle_13 |0, 0\rangle_24 \varphi(R_c) \times \frac{1}{\sqrt{2}} \left[ \psi'_2(r_{13}, r_{2} - R_{13})\psi'_3(R_{123} - r_{4}) + \psi'_2(r_{13}, r_{4} - R_{13})\psi'_3(R_{123} - r_{2}) \right]. \tag{19}
\]

Here \(R_{ijk}\) is the center-of-mass coordinate of particles \(i, j,\) and \(k\). In Eq. (17), \(\psi'_0\) describes the internal motion of the proton pair and of the neutron pair, while \(\psi'_1\) describes the relative motion of the two pairs in a \(p\) wave. In Eq. (18), \(\psi_2\) describes the internal motion of a triton, while \(\psi_3\) describes the relative motion of the triton and a proton in a \(p\) wave. In Eq. (19), \(\psi'_2\) describes the internal motion of a \(^2\)He\(^3\) nucleus, while \(\psi'_3\) describes the relative motion of the \(^2\)He\(^3\) and a neutron in a \(p\) wave. Note that \(\Psi_2, \Psi_3, \Psi_4, \Psi_5\) all have identical values of \(L, S\) and parity, namely 1, 0, -1, respectively. In general, there will be some wavefunction overlap between \(\Psi_2\) and each of \(\Psi_3, \Psi_4, \Psi_5\).

In channels 1 and 2, the emission of an M1 photon converts \(\Psi_1\) into the intermediate virtual state \(\Psi_3\). Unlike \(\Psi_1, \Psi_3\) probably has a large energy linewidth. Nevertheless, the mean energy of \(\Psi_3\) is similar to that of \(\Psi_1\). This suggests that, on the average, the M1 photon takes up a rather small portion of the available reaction energy \(Q_1 = 23.846\) MeV. In channel 1 the final state is achieved by emitting an E1 photon, with energy close to \(Q_1\), thereby removing the orbital angular momentum \(L = 1\), allowing the final ground state to have \(L = 0\). The energy of a 24 MeV gamma ray is on the high side for absorption by Compton scattering and beyond the threshold for absorption by pair creation [10]. The absorption mean free path of such a photon is \(5.3 \times 10^4\) cm in air, 64 cm in water, 1.9 cm in palladium metal. The absence of experimental observation of such a photon suggests that channel 1 is an unlikely mode of decay. I will assume that it is the least likely of the four channels.

In channel 2 the final state is achieved by the \(\alpha\) particle being in a free-particle \(p\)-wave state with kinetic energy close to \(Q_2\). The recoil momentum is taken up by the palladium metal as a whole. In this sense, channel 2 is analogous.
to the Mössbauer effect, only there the recoil momentum counterbalances that of a photon rather than an α particle. What is the mechanism that allows the internal energy of the intermediate state to be converted to center-of-mass kinetic energy in the final state? The answer is that there is a coupling between the internal motion and the center-of-mass motion. Consider the harmonic-oscillator potential well in a tetrahedral cavity in palladium metal. A deuteron in such a cavity experiences a force having an associated spring constant of \( k = m_D \omega_D^2 \). The antiparallel-spin proton-pair composite particle of the intermediate state thus has a spring constant \( 2m_D \omega_D^2 \) (because of the charge +2e), while the antiparallel-spin neutron-pair composite particle has zero spring constant (because of zero charge). (As before, I am here making the approximation that neutrons and protons have the same mass.) The Hamiltonian for the harmonic-oscillator forces now contains a term

\[
H' = m_D \omega_D^2 R_c \cdot (R_{13} - R_{24}).
\]  

This is the coupling between internal and center-of-mass motion that allows decay via channel 2. I believe that in experiments where very large amounts of heating are observed this channel is the major source of the heat. The energy-loss mean free path of a 24 MeV α particle in palladium metal is approximately \( 4.5 \times 10^{-2} \text{ cm} \).

In channel 3, the emission of an M1 photon converts \( \Psi_1 \) into the final state \( \Psi_4 \) involving a triton and a proton. In channel 4, the emission of an M1 photon converts \( \Psi_1 \) into the final state \( \Psi_5 \) involving \( ^2\text{He}^3 \) and a neutron. In each of these two channels, the outgoing massive particles must be in a relative \( p \) state in order to preserve orbital angular momentum \( L \). Thus there is a centrifugal barrier in each channel of value

\[
E_L = \frac{8\hbar^2}{3m_D R^2} = 6.161 \text{ MeV}.
\]  

(Here I have used the approximation that the reduced mass is \( (3/8)m_D \) in both cases.) In channel 3 there is added to this the Coulomb barrier

\[
E_C = \left( \frac{e^2}{R} \right) = 0.480 \text{ MeV}.
\]  

In this channel, the minimum possible height of barrier through which \(^1\text{H}^3\) and \( p \) must tunnel is

\[
E_L + E_C - Q_3 = 2.608 \text{ MeV}.
\]  

In channel 4, the minimum possible height of barrier through which \(^2\text{He}^3\) and \( n \) must tunnel through is

\[
E_L - Q_4 = 2.892 \text{ MeV}.
\]  

We have the remarkable result that the tunneling process is more difficult in channel 4 despite the absence of a Coulomb barrier. It is consistent with experimental observations [11] of tritium/neutron ratios as high as \( 10^8 \). In actuality,
the barrier heights will have values somewhat larger than those of Eqs. (23) and (24) since some of the reaction energy $Q$ must be shared with the emitted magnetic-dipole photon, but the energy of the photon will be quite small in order not to quench the tunneling process. I believe that because of this tunneling process channels 3 and 4 are less likely than channel 2. As in channel 2, there is a term coupling internal and center-of-mass motion in channel 4. However, the tunneling process is optimized by allowing no transfer of energy from internal to center-of-mass motion.

Several experiments [12] have indicated a correlation between the production of large amounts of heat and the production of $^2\text{He}^4_4$. This is consistent with channel 2 being the primary mode of decay. During such an experiment there should be a mixture of normal $^2\text{He}^4_4$ and the isomer $^2\text{He}^{4*}$ present. The latter is readily distinguishable from the former by the presence of a nuclear magnetic moment. By using nuclear magnetic resonance to test for the presence of the isomer, one should be able to check for the correctness (or falsity) of the present scenario.

The probability per unit time the two deuterons in a tetrahedral cavity will fuse is governed by the general expression

$$\lambda = A |\Psi(R)|^2,$$

where $\Psi(R)$ is the value of the two-deuteron wavefunction at the boundary of the nuclear interaction region. In the case of nonresonant tunneling,

$$A = 1.478 \times 10^{-16} \text{ cm}^3/\text{sec}. \quad (26)$$

In their most recent paper [1], Parmenter and Lamb calculated a value

$$|\Psi(R)|^2 = 6.208 \times 10^{-8} \text{ cm}^{-3}, \quad (27)$$

resulting in

$$\lambda = 9.176 \times 10^{-24} \text{ sec}^{-1}. \quad (28)$$

In the case of resonant tunneling described in the present paper, we must replace $A$ by

$$A' = \frac{4}{3} \pi R^3 T^{-1}, \quad (29)$$

where $T$ is the decay lifetime of the isomer $^2\text{He}^{4*}$. I assume

$$T = 2.592 \times 10^5 \text{ sec (3 days)}, \quad (30)$$

based on the experimental experience that large amounts of excess heat are generated only after running the fusion cells for long periods of time. Note that

$$A' = 4.352 \times 10^{-43} \text{ cm}^3/\text{sec} \quad (31)$$
is smaller than \( A \) by a factor of \( 10^{-27} \). When resonant tunneling is occurring, the wavefunction in the nuclear interaction region increases by a small amount each time the two deuterons bang into the Coulomb barrier separating them (with the characteristic angular frequency \( \omega_0 = 4.456 \times 10^{14} \text{sec}^{-1} \)). Thus \( \Psi(R) \) should be replaced by

\[
\Psi'(R) = (\omega_0 T/2\pi)\Psi(R)
\]

in Eq. (25). This replacement is justified as long as

\[ \frac{4}{3} \pi R^3 |\Psi'(R)|^2 << 1, \]  

a condition that is invariably satisfied. For \( p \)-wave resonant tunneling, the value of \( \Psi(R) \) will be less than that of Eq. (27), since the Gamow penetration factor is now appropriate to a combined Coulomb-centrifugal barrier. I estimate this reduces \( \Psi(R) \) by a factor of \( 10^{-2} \) at most, since the centrifugal portion of the barrier is negligible over most of the tunneling range of the zero-energy deuteron pairs. Putting in the numbers, we get

\[
\lambda' = A' |\Psi'(R)|^2 = 9.129 \times 10^{-13} \text{sec}^{-1}. \]

\( \lambda' \) is greater than \( \lambda \) by a factor of \( 10^{11} \), more than large enough to explain the amount of excess heat seen in any experiment. It should be admitted that in Eq. (32) I assumed that the resonant tunneling of a pair of deuterons went on undisturbed for the full lifetime of the isomer (3 days). In actuality, thermal motion of the lattice may interrupt the process in much shorter times.

The equilibrium separation distance of a deuteron pair inside a tetrahedral cavity in palladium metal is [1]

\[ R_m = 0.37709 \times 10^{-8} \text{cm.} \]  

Considering the pair as a rigid rotator, the excitation energy of the \( p \)-state of the rotator is

\[ (2\hbar^2/m_D R_m^2) = 2.9199 \times 10^{-2} \text{eV}. \]

This corresponds to a temperature of 66°C Celsius. Thus there is a reasonable possibility of the deuteron pair being in the \( p \) state at room temperature.

In order for resonant tunneling to occur, the energy of the deuteron pair must closely match that of the \(^2\text{He}^{4*}\) isomer. The very great difficulty in obtaining reproducible results experimentally may well be a consequence of the difficulty in matching these energies. It has been observed experimentally [13] that the use of a pulsed mode of operation increases the production of tritium and excess heat. The pulsing may allow for energy matching on a periodic basis.

In conclusion, we have seen that the assumption of resonant tunneling of the Coulomb barrier separating two deuterons can lead to simple explanations of several surprising experimental observations: the very large tritium/neutron
ratio's, the association of excess heat with $^2\text{He}^4$ production, the difficulty in obtaining reproducible results. The validity of the model can be checked by looking for the nuclear magnetic moment of the nuclear isomer.

I am greatly indebted to Robert W. Bass for renewing my interest in the process of resonant tunneling.
References


Note added (July 1998): I wish to thank Louis Brown for pointing out that decay channels 3 and 4 can each occur without emission of a magnetic dipole photon. In each channel it is necessary only that the two emerging fermions have their spins parallel rather than anti-parallel. The argument remains unchanged that the tritium/neutron is very large. Also channel 2 can occur without emission of a magnetic dipole photon provided that a conduction electron flips its spin when helium isomer decays to ground state. The absence of magnetic dipole radiation will increase the probability of decay in all three channels.