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ARE THERE SOME LOOSE BOUND STATES OF NUCLEUS-NUCLEUS TWO-BODY SYSTEM?

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ABSTRACT

We obtain the possible solutions of the stable Schrödinger equation with Coulomb barrier and square well potential induced by nuclear force for a two nuclei system. The wave functions, energy eigenvalues and the existence condition of the possible loose bound states of this system are given. The binding energy is few keV for ground states of some light nuclei here.

1. INTRODUCTION

The nuclear reactions, that any nucleon is added by radioactive capture or radioactivity, are of central importance to understand the nuclear structure. Any one, who is interested in investigating the structure of nucleons and their properties, should be excited to the progress on experimental results about the emission of charged particles, such as clusters (specially heavy clusters-^{14}C, O, F, Ne, Mg and Si), fission isomers, etc. after the first case of radioactive proton emission was observed in 1970 [1]. The discovery of ^{14}C radioactivity gave particularly rise to great excitement in nuclear physics [2]. After ^{14}C radioactivity discovered neon radioactivity was discovered in 1985 [3], magnesium in 1987[4]. Evidence for silicon was obtained in 1989[5]. Fluorine and oxygen were discovered in 1992 and 1993, respectively [6,7]. Furthermore, there are another experimental results from Miley et al [8], which should give rise to great excitement in nuclear physics also. That is the observation results of transmutation products in electrode during the electrolysis of water. These results may be explained by assuming that the nucleus or the cluster is a system of independent particles, moving in a mean filed created by all nucleons, or by the theory, which could allow us to treat two-body breakup in fission. Here, it may be valuable to do it on traditional path, as a general two-body system treatment. In this case, it may be obtained the possible solutions of the negative energy bound states of the stable Schrödinger equation with Coulomb potential barrier and square well potential induced by nuclear force for the system of two nuclei [9,10].

2. MODEL AND RESULTS

Consider a general spherical square well combined with the Coulomb potential for a two nucleus- nucleus system:

$$U(r) = \begin{cases} Z_1 Z_2 e^2 / r & (r > r_0) \\ -U_0 & (r < r_0) \end{cases} \tag{1}$$

A regular solutions of the radial equation is:

$$u''(r) + \left\{ \frac{2\mu}{\hbar^2} \left[E - U(r) \right] - \frac{l(l+1)}{r^2} \right\} u(r) = 0 \tag{2}$$

We will discuss its solutions by three regions.

(A) In the $r > r_0$ region:

$$u''(r) + \left\{ \frac{2\mu}{\hbar^2} \left[E - \frac{Z_1 Z_2 e^2}{r} \right] - \frac{l(l+1)}{r^2} \right\} u(r) = 0 \tag{3}$$

This equation is equivalent to:

$$u''(\rho) + \left\{ -\frac{\beta}{\rho} - 4^{-1} - \frac{l(l+1)}{\rho^2} \right\} u(\rho) = 0 \tag{4}$$

for $E < 0$ with $\alpha = \sqrt{8\mu |E| / \hbar^2} > 0$, $\beta = (Z_1 Z_2 e^2 / \hbar)\sqrt{\mu / |2E|} > 0$ and $\rho = \alpha r$.

1. For $\rho \to \infty$, the radial function behaves as $\exp(\rho/2)$ or $\exp(-\rho/2)$. Of course, it should be the latter one for a suitable result.

2. On the other hand, it behaves as ρ^{l+1} and ρ^{-l} for $\rho \to \infty$. For ρ^{l+1} we have:

$$u(\rho) = \rho^{l+1} \exp(-\rho/2) \sum_{\nu=0}^{\infty} b_\nu \rho^\nu \tag{5}$$

No any solution exists in this case. For ρ^{-l} we have:

$$u(\rho) = \rho^{-l} \exp(-\rho/2) \sum_{\nu=0}^{\infty} b_\nu \rho^\nu \tag{6}$$

with the recursion relation

$$b_{\nu+1} = \frac{\nu - l + \beta}{(\nu - l + 1)(\nu - 1) - l(l+1)} b_\nu \tag{7}$$

and

$$\beta = l - n_r = n, \quad n = 1,2,3...... \tag{8}$$

$$E_n = -\frac{Z_1^2 Z_2^2 \mu e^4}{2\hbar^2 n^2}, \ n = 1,2,3...... \tag{9}$$

$$l = n + n_r = n, n+1, n+2...... \ (l \geq n, \ l \neq 0) \tag{10}$$

For $n_r = l - n$, the wave function of the corresponding bound state is the (un-normalized) wave function constructed with the radial solution:

$$u_{n,l}(r) = (\alpha_n r)^{-l} \exp(-\frac{\alpha_n r}{2}) \sum_{\nu=0}^{l-n} b_\nu (\alpha_n r)^\nu, \ r > r_0 \tag{11}$$

(B) In the $r < r_0$ region:

$$u''(r) + \left\{ k^2 - \frac{l(l+1)}{r^2} \right\} u(r) = 0 \tag{12}$$

with

$$k = \sqrt{\frac{2\mu(U_0 - |E|)}{\hbar^2}} > 0 \tag{13}$$

for $u(0) = 0$ and

$$u(r) = C \bullet j_l(kr) \tag{14}$$

(C) At $r = r_0$, values of U_0, r_0, E_n, Z_1, Z_2, μ, n and l can be obtained from the boundary condition:

$$\frac{\alpha_n \left[u'_{n,l}(\alpha_n r_0) \right]}{u_{n,l}(\alpha_n r_0)} = \frac{k_n \left[j'_l(k_n r_0) \right]}{j_l(k_n r_0)} \tag{15}$$

We have:

$$u_{1,1}(r) = \begin{cases} \frac{b_0 a_\mu}{2r} \exp(-\frac{r}{a_\mu}), & r > r_0 \\ C\left[\frac{\sin(k_1 r)}{k_1 r} - \cos(k_1 r) \right], & r < r_0 \end{cases} \tag{16}$$

where $a_\mu = a_p/(\mu Z_1 Z_2)$, $\mu = A_1 A_2/(A_1 + A_2)$, $a_p = h^2/(4 m_p \pi^2 e^2) = a_e/1840 \approx 30$ fm, $a_e = 5 \times 10^4$ fm.

For $n = 1$, $E = E_1 = -2\pi^2 Z_1^2 Z_2^2 \mu e^4/(n^2 h^2)$ or

$$E_1 = -\frac{Z_1^2 Z_2^2 \mu e^4}{2\hbar^2} = 25 Z_1^2 Z_2^2 \mu \text{ (keV)} \tag{17}$$

Therefore

$$\frac{1}{k_1 r_0} + k_1 a_\mu = \cos(k_1 r_0) \tag{18}$$

When $a_\mu/r_0 >> 1$, we get:

$$k_1 r_0 \approx i\pi,\ i = 1,2,3... \tag{19}$$

or

$$U_0 - |E_1| = 225\frac{i^2}{\mu r_0^2},\ i = 1,2,3... \tag{20}$$

The possible energy levels of the stable Schrödinger equation with Coulomb barrier and square well potential induced from nuclear force for two nuclei system could be obtained: $|E_1| = 25$ keV for D-D; $|E_1| = 16.7$ keV for p-D; $|E_1| = 30$ keV for D-T and $|E_1| = 12.5$ keV for p-p system. Of course, the light-heavy nuclei systems also have the similar results.

3. CONCLUSION

Based on the model of two nuclei system, our results indicate a kind of loose bound state may exist and it has energy level in the range of a few 10 keV, which may be origins of X-ray observed in experiments.

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