

POSSIBLE EVIDENCE OF COLD D(D,p)T FUSION FROM DEE'S 1934 EXPERIMENT

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ABSTRACT

$D(D,p)T$ fusion probabilities for the back-to-back proton-tritium tracks observed in Dee's 1934 experiment are calculated using the conventional theory and found to be many orders of magnitude smaller than those inferred from Dee's data. Our results indicate that Dee's data may be evidence for cold fusion, possibly due to low-energy reaction barrier transparency as recently proposed. Therefore it is important to repeat Dee's experiment with modern facilities.

1. INTRODUCTION

In 1934, Oliphant, Harteck, and Lord Rutherford^{1,2} reported the discovery of deuterium-deuterium fusion via nuclear reactions, $D(D,p)T$ and $D(D,n)^3He$. They bombarded deuterated ammonium chloride (ND_4Cl), ammonium sulphate ($(ND_4)_2SO_4$) and orthophosphoric acid (D_3PO_4) with 20 ~ 200 keV deuterium (called "diplogen" then) ion (D^+) beam generated from a Cockcroft-Walton discharge tube.³ Later in the same year (1934), Dee⁴ studied the nuclear reaction $D(D,p)T$ more carefully using a 160 keV D^+ beam on a $(ND_4)_2SO_4$ target, and photographed ionization tracks of p and T in a cloud chamber. Occasionally, proton-tritium ($p-T$) pairs were observed with the angle between the tracks very near to 180° .⁵ Dee⁵ attributed these tracks to $D(D,p)T$ reactions involving deuterons which have lost energy by collisions in the target. The expansion chamber detection system used by Dee^{4,5} was developed earlier by Dee and Walton.⁶ The D^+ beam generation system used by Dee was an improved Cockcroft-Walton³ discharge tube constructed by Oliphant and Rutherford⁷ which generated a higher D^+ beam current (~ 100 μA). Recently, Fleischmann⁸ suggested that these results obtained by Dee⁵ are the first indication that there are low energy fusion channels in solid lattices as in the case of the cold fusion electrolysis experiments.⁹ Fleischmann's suggestion was criticized by Close¹⁰ on two grounds. The first is that Dee's photographs do not show that the tracks are exactly back-to-back and hence one cannot eliminate the possibility that the incident deuteron had even one keV of energy, which is comparable to the solar core temperature. The second objection by Close (and also by Petrasso¹⁰) was that no energetic tritium or proton had been observed in the cold fusion electrolysis experiments with deuterated palladium. More recently, Huizenga¹¹ objected to Fleischmann's statement⁸ that a *significant number* of back-to-back tracks were observed by Dee, since what Dee⁵ actually stated was that *occasionally* back-to-back $p - T$ tracks were observed.

In this paper, Dee's results are analyzed using the conventional theory to establish whether the back-to-back $p - T$ tracks observed by Dee⁵ suggest an anomalous effect, in order to resolve the controversy between Fleischmann's suggestion⁸ and its objections by Close,¹⁰ Petrasso,¹⁰ and Huizenga.¹¹

2. FUSION KINEMATICS

When D^+ ions (deuterons) are incident on the $(ND_4)_2SO_4$ target, the dominant fusion reactions are known to be



and



for an incident deuteron laboratory (LAB) kinetic energy E_D greater than $\sim 10 \text{ keV}$,¹² with the Q values of 4.033 MeV and 3.269 MeV, respectively. For $D(D,p)T$, eq. (1), the velocities \vec{v}_p and \vec{v}_T of the emitted proton (p) and tritium (3H or T) are co-planar with the velocity \vec{v}_D of the incident deuteron. The scattering angles, θ_L^p and θ_L^T , of the emitted p and T , in the LAB frame, are measured from the direction of the incident deuteron velocity \vec{v}_D (which is the same as the direction of $D + D$ center-of-mass (CM) velocity, \vec{v}_C) i.e., $\cos \theta_L^p = \vec{v}_D \cdot \vec{v}_p = \vec{v}_C \cdot \vec{v}_p$ and $\cos \theta_L^T = \vec{v}_D \cdot \vec{v}_T = \vec{v}_C \cdot \vec{v}_T$. In the CM frame, the directions of p and T velocities, \vec{v}'_p and \vec{v}'_T , are opposite and the p and T scattering angles in the CM frame, θ_C^p and θ_C^T , as measured from \vec{v}_C add up to 180° , $\theta_C^p + \theta_C^T = 180^\circ$. The proton scattering angles, θ_L^p and θ_C^p , are related by¹³

$$\tan \theta_L^p = \frac{\sin \theta_C^p}{\gamma_p + \cos \theta_C^p} \quad (3)$$

with

$$\gamma_p = \left[\frac{m_T(m_p + m_T)}{m_D m_p} \left(\frac{Q_1}{E_D} \right) + \frac{m_T(m_T + m_p - m_D)}{m_D m_p} \right]^{-\frac{1}{2}} \quad (4)$$

while the triton scattering angles, θ_L^T and θ_C^T , are related by

$$\tan \theta_L^T = \frac{\sin \theta_C^T}{\gamma_T + \cos \theta_C^T} \quad (5)$$

with

$$\gamma_T = \left[\frac{m_p(m_T + m_p)}{m_D m_T} \left(\frac{Q_1}{E_D} \right) + \frac{m_p(m_p + m_T - m_D)}{m_D m_T} \right]^{-\frac{1}{2}} \quad (6)$$

where m_p , m_D , and m_T are the rest masses of proton, deuteron, and tritium, respectively.

For the special case of $\theta_C^p = \theta_C^T = 90^\circ$, eqs. (3) and (5) reduce to

$$\tan \theta_L^p = \frac{1}{\gamma_p} = [5.979(Q_1 / E_D) + 2.986]^{1/2} \quad (7)$$

and

$$\tan \theta_L^T = \frac{1}{\gamma_T} = [0.6676(Q_1 / E_D) + 0.3334]^{1/2} \quad (8)$$

The calculated values of θ_L^p , θ_L^T , and $\theta_L^{pT} = \theta_L^p + \theta_L^T$ for the case of $\theta_c^p = \theta_c^T = 90^\circ$ are listed in Table 1 for several selected values of E_D . From the description and pictures given in references 5 and 6, Dee's Wilson expansion chamber has acceptance angles of $\sim 30^\circ$, i.e., $75^\circ \lesssim \theta_L^p \lesssim 105^\circ$ and $75^\circ \lesssim \theta_L^T \lesssim 105^\circ$. Within the above acceptance angles, the results of $\Delta\theta = 180^\circ - \theta_L^{pT}$ listed in Table 1 decrease only by $\sim 5\%$ for $E_D \leq 10$ keV. Therefore, if the accuracy of Dee's measurements of θ_L^{pT} is $\sim \pm 1^\circ$, we expect that $E_D \leq 2$ keV for the back-to-back pT tracks with $\Delta\theta \leq 2^\circ$, while, for an accuracy of $\pm 2.0^\circ$ for θ_L^{pT} , we expect $E_D \lesssim 10$ keV, corresponding to $\Delta\theta \leq 4^\circ$.

3. CONVENTIONAL FUSION PROBABILITY AND RATE

The probability $P(E_i)$ for a deuteron with the initial LAB kinetic energy E_i to undergo the fusion reaction (1) while slowing down in a deuterated ammonium sulfate $(ND_4)_2SO_4$ target can be written as

$$P(E_i) = \int dx n_D \sigma(E_{DD}) = \int_0^{E_i} dE_D \frac{n_D \sigma(E_{DD})}{|dE_D / dx|} \quad (9)$$

where n_D is the target deuteron number density, $\sigma(E_{DD})$ is the cross-section for reaction (1), dE/dx is the stopping power for deuteron by the target atoms, and E_D and E_{DD} are the deuteron kinetic energies in the LAB and CM frames, respectively ($E_{DD} = E_D/2$).

3.1. Stopping Power

The stopping power for deuteron by the target atom j with the density n_j can be taken from Ref. 14. For a deuteron laboratory kinetic energy $E_D \leq 20$ keV, it is given by¹⁵

$$\frac{dE_D}{dx} = n_j A_1 (E_D / 2)^{1/2} \times 10^{-18} \text{ keV} - \text{cm}^2 \quad (10)$$

With E_D in units of keV. For $20 \text{ keV} \leq E_D \leq 1 \text{ MeV}$, we have

$$\left[\frac{dE_D}{dx} \right]^{-1} = \left[\frac{dE_D}{dx} \right]_{slow}^{-1} + \left[\frac{dE_D}{dx} \right]_{high}^{-1}, \quad (11)$$

where explicit expressions for $[dE/dx]_{slow}$ and $[dE/dx]_{high}$ are given in reference 15. Since $[dE_D/dx]$ (eq.(11)) $\sim [dE_D/dx]_{slow}$ and $[dE_D/dx]_{slow}$ agrees with dE_D/dx given by eq. (10) within a factor of 2 or less for $20 \text{ keV} < E_D < 160 \text{ keV}$, it is fairly accurate to assume that the stopping power is given by eq. (10) for $E_D < 160 \text{ keV}$. The atom number densities n_j for the mixtures of $\alpha(NH_4)_2SO_4 + \beta(ND_4)_2SO_4$ with $\alpha + \beta = 1$ are $n_H = \alpha(6.45 \times 10^{22}/\text{cm}^3)$, $n_D = \beta(6.45 \times 10^{22}/\text{cm}^3)$, $n_N = 1.61 \times 10^{22}/\text{cm}^3$, $n_O = 3.22 \times 10^{22}/\text{cm}^3$, and $n_S = 0.806 \times 10^{22}/\text{cm}^3$, respectively for hydrogen deuterium, nitrogen, oxygen and sulfur, respectively. Using eq. (10), the stopping powers for deuteron by the target atoms, H, D, N, O, and S are given by

$$\frac{dE_D}{dx}(H) = \alpha(5.756 \times 10^4)\sqrt{E_D} \text{ keV/cm} \quad (12)$$

$$\frac{dE_D}{dx}(D) = \beta(5.756 \times 10^4)\sqrt{E_D} \text{ keV/cm} \quad (13)$$

$$\frac{dE_D}{dx}(N) = 3.363 \times 10^4\sqrt{E_D} \text{ keV/cm} \quad (14)$$

$$\frac{dE_D}{dx}(O) = 6.038 \times 10^4\sqrt{E_D} \text{ keV/cm} \quad (15)$$

and

$$\frac{dE_D}{dx}(S) = 1.965 \times 10^4\sqrt{E_D} \text{ keV/cm}, \quad (16)$$

respectively. Therefore, the stopping power for D is the target consisting of a $\alpha(NH_4)_2SO_4$ and $\beta(ND_4)_2SO_4$ ($\alpha + \beta = 1$) is given by the sum of the eqs. (12) through (16),

$$\frac{dE_D}{dx} = 1.712 \times 10^5\sqrt{E_D} \text{ keV/cm} \quad (17)$$

where E_D is in units of keV.

3.2. Parameterized Cross-Section

The cross-section $\sigma(E_{DD})$ for the $D(D,p)T$ reaction has not been measured for $E_{DD} \lesssim 5 \text{ keV}$. For $E_{DD} \lesssim 5 \text{ keV}$, $\sigma(E_{DD})$ is calculated by extrapolating the experimental values of $\sigma(E_{DD})$ at higher energies using the parameterization ($E = E_{DD}$)

$$\sigma(E) = \frac{S(E)}{E} T_G(E) \quad (18)$$

where $T_G(E) = \exp[-(E_G/E)^{1/2}]$, $E_G = (2\pi\alpha Z_D Z_D)^2 \mu c^2$ or $E_G^{1/2} \approx 31.39 (keV)^{1/2}$ with the reduced mass $\mu = m_D m_D / (m_D + m_D) = \frac{1}{2} m_D$. The transmission coefficient (“Gamow” factor) $T_G(E)$ results from the approximation $E \ll B$ (Coulomb barrier height). Note that $\sigma(E)$ described by eq. (18) is valid only from non resonance fusion reactions. The S-factor, $S(E)$, is extracted from the experimentally measured values¹² of the cross-section, $\sigma(E)$, for $E \gtrsim 4$ keV and is nearly constant,¹² $S(E) \approx S(0) = 52.9 keV - barn$, for both reactions (1) and (2).

3.3. Conventional Estimates of Fusion Probability

Using the results of eqs. (17) and (18) with $n_D = \beta(6.45 \times 10^{22} / cm^3)$, $P(E_i)$ given by eq. (9) can be written as

$$\begin{aligned} P(E_i) &= \int_0^{E_i} dE_D \frac{n_D S(E_{DD})}{|dE_D/dx| E_{DD}} e^{-\sqrt{E_G}/\sqrt{E_{DD}}} \\ &= \frac{2n_D S(0)}{1.712 \times 10^4 (keV/cm)} \frac{1}{\sqrt{E_G'}} e^{-\sqrt{E_G'}/\sqrt{E_i}} \end{aligned}$$

where $\sqrt{E_G'} = \sqrt{2}\sqrt{E_G} = 44.39\sqrt{keV}$ and E_i is in units of keV. Using $S(0) = 52.9 \times 10^{-24} cm^2 - keV$ and $n_D = \beta(6.45 \times 10^{22} / cm^3)$, $P(E_i)$ can be written as

$$P(E_i) = \beta(0.90 \times 10^{-6}) e^{-\sqrt{E_G'}/\sqrt{E_i}} \quad (20)$$

The calculated values of $P(E_i)$ using eq. (20) with $\beta = 1$ are listed in Table 2 for several selected values of E_i .

3.4. Conventional Estimates of Fusion Rates

As described by Dee and Walton,⁶ Dee⁵ used a discharge tube similar to that of Oliphant and Rutherford⁷ which gave a proton beam current of 100 μA using an accelerating voltage not greater than 250 kV. Assuming that the same system generated the deuteron beam current of $I = 100 \mu A$ with a deuteron LAB energy of $E_i = 160$ keV, we can obtain conventional estimates of the expected fusion rates $R(E_i)$ from

$$R(E_i) = \Phi P(E_i) \quad (21)$$

where Φ is the incident deuteron flux given by

$$\Phi = (0.625 \times 10^{19} D^+ / sec) I \quad (22)$$

with I in units of amperes. For $I = 100 \mu A$, the conventional estimate of $R(E_i)$ is

$$R(E_i) = 0.562 \times 10^9 \beta e - \sqrt{E_G} / \sqrt{E_i} / \text{sec} \quad (23)$$

with E_i in units of keV. The calculated values of $R(E_i)$ from eq. (23) with $\beta = 1$ are listed in Table 2 for several selected values of E_i . It is likely that Dee used a much lower current than 100 μA by controlling it with a beam shutter in order to have a manageable counting rate for the pT tracks produced in his Wilson expansion chamber.

4. Analysis of Dee's Data

From the conventional estimates of $P_R(E_i)$ given in Table 2, we see that out of a total of 10^{12} pT tracks ($162^\circ < \theta_L^{pT} < 180^\circ$) produced, only one is expected to be a back-to-back ($178^\circ < \theta_L^{pT} < 180^\circ$) pT track, which is impossible to be observed occasionally with Dee's experimental set up. If we interpret Dee's "occasional observations" to mean 1 out of 100 (a reasonable interpretation) which corresponds to a degraded deuteron kinetic energy of $E_D = 30$ keV (see Table 2), we expect $\theta_L^{pT} \gtrsim 172^\circ$ (see Table 1), which cannot be back-to-back pT track with the accuracy of $\pm 2^\circ$ for measuring θ_L^{pT} .

4.1. Scattered Deuteron Mechanism

We now investigate a mechanism in which the incident deuteron could be scattered by a target atom into the Wilson expansion chamber acceptance angles, $\theta_D = 90^\circ \pm 15^\circ$, prior to fusing with a target deuteron to produce a back-to-back pT track (which is possible if θ_c^p or θ_c^T is nearly parallel to θ_D).

For a screened Coulomb potential

$$V_c(r) = \frac{Z_D Z_j e^2}{r} e^{-r/a} \quad (24)$$

with a screening radius a , and atomic numbers $Z_D (=1)$ and Z_j for the deuteron and target atom j , respectively, the scattering amplitude $f(\theta_c)$ in the Born approximation is given by¹⁶

$$f(\theta_c) = \frac{2 \mu Z_j e^2 a^2}{\hbar^2 (4k^2 a^2 \sin^2 \frac{\theta_c}{2} + 1)} \quad (25)$$

where $\hbar^2 k^2 / 2 \mu = E_{CM}$ with reduced mass μ and the CM kinetic energy E_{CM} . The probability of the incident deuteron being scattered by a target atom j into the CM angle θ_c is then

$$P_j(\theta_c) = \left| \frac{f(\theta_c)}{f(0)} \right|^2 = \left[8 \left(\frac{m_D}{\hbar^2} \right) \left(\frac{m_D}{m_D + m_j} \right)^2 E_{D^{\circ 2}} \left(\sin \frac{\theta_c}{2} \right)^2 + 1 \right]^{-2} \quad (26)$$

where E_D and a are in units of keV and \AA , respectively. The CM angle θ_c is related to the deuteron LAB scattering angle θ_L by

$$\tan \theta_L = \frac{\sin \theta_c}{(m_D / m_j) + \cos \theta_c} \quad (27)$$

Using $a = \hbar^2 / m_e e^2 Z_j^{1/3} = 0.529 \text{ \AA} / Z_j^{1/3}$ and eq. (27), $P_S(\theta)$ for deuteron-sulfur atom Coulomb scattering for $\theta_c = 30^\circ$ or $\theta_L = 28.3^\circ$ is calculated and found to be $P_S(30^\circ) = 2.32 \times 10^{-8}$. Since $P_D(30^\circ)$, $P_N(30^\circ)$, and $P_O(30^\circ)$ are smaller than $P_S(30^\circ)$, we can conclude that only a few out of 10^8 incident 160 keV deuterons move out beyond $\theta_L \approx 28^\circ$ of the incident direction after the first encounter with the target atom. Therefore, the scattered deuteron mechanism cannot explain Dee's back-to-back pT tracks.

4.2. Suggested Experimental Tests

Since the conventional estimates of fusion probability and rate for the events observed by Dee⁵ with the $p - T$ opening angle $180^\circ > \theta_L^{pT} > 178^\circ$ corresponding to the deuteron LAB kinetic energy $E_D \lesssim 2 \text{ keV}$ are smaller by many orders of magnitude than the inferred values from Dee's experiment, it suggests strongly that the conventional estimates are not reliable at a low energies, $E_D < 2 \text{ keV}$. It is therefore important to repeat Dee's experiment with improved Wilson expansion chambers. In addition to using the Wilson expansion chamber, one should also use other modern visual detecting systems¹⁷ such as a diffusion cloud chamber, with high current (continuous or pulsed) low-energy deuteron beams.

5. CONCLUSION

Contrary to objections raised by Close¹⁰, Petrasso¹⁰, and Huizenga¹¹, the suggestion made by Fleischmann that Dee's back-to-back pT tracks are the first indication of cold fusion may have validity since it is shown that the conventional theoretical estimates cannot explain the back-to-back pT tracks observed by Dee⁵.

One plausible explanation¹⁸ of Dee's data for the back-to-back pT tracks, based on a general and more realistic solution of the transmission coefficient $T_{KZ}(E)$ by Kim and Zubarev¹⁸, is that reaction barrier transparency exists for the transmission coefficient near the fusion threshold energy.¹⁹ The conventional Gamow transmission coefficient, $T_G(E)$, is restricted to non-resonant reactions, and hence cannot describe such resonant behavior of transmission coefficient. In other conventional theoretical models, Breit-Wigner (BW) resonances are included in the S-factor, $S(E)$, in eq. (18), but any enhancement of $\sigma(E)$ due to the BW resonance is limited to at most a few orders of magnitude increase and hence cannot explain Dee's data. Therefore, it is important to repeat Dee's data. Therefore, it is important to repeat Dee's experiment with modern facilities and techniques.

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Table 1

Proton and tritium scattering angles for selected incident deuteron kinetic energies in the LAB frame assuming the CM scattering angles $\theta_C^p = \theta_C^T = 90^\circ$.

$E_k(\text{keV})$	θ_L^p	θ_L^T	$\theta_L^{pT} = \theta_L^p + \theta_L^T$	$\Delta\theta = 180^\circ - \theta_L^{pT}$
1	89.63	88.90	178.53	1.43
2	89.48	88.44	177.92	2.08
3	89.36	88.09	177.45	2.55
5	89.18	87.53	176.71	3.29
10	88.83	86.52	175.35	4.65
20	88.35	85.08	173.43	6.57
30	87.98	83.99	171.97	8.03
160	85.39	76.43	161.82	18.18

Table 2

Fusion probabilities $P(E_i)$, relative probabilities $P_R(E_i)$ (normalized with $P(160 \text{ keV}) = 1$), and fusion rates $R(E_i)$, with the incident deuteron current of $100 \mu\text{A}$, for incident deuteron LAB kinetic energies, E_i . $\beta = 1$ is assumed.

$E_i(\text{keV})$	$P(E_i)$	$P_R(E_i)$	$R(E_i)(\text{sec}^{-1})$
1	0.47×10^{-25}	1.76×10^{-18}	2.96×10^{-11}
2	2.10×10^{-20}	0.78×10^{-12}	1.31×10^{-5}
3	0.66×10^{-17}	2.47×10^{-10}	0.42×10^{-2}
5	2.15×10^{-15}	0.80×10^{-7}	1.34
10	0.72×10^{-12}	2.68×10^{-5}	0.45×10^3
20	0.44×10^{-10}	1.63×10^{-3}	2.74×10^4
30	2.72×10^{-10}	1.01×10^{-2}	1.70×10^5
160	2.69×10^{-8}	1.0	1.68×10^7