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TRITIUM PRODUCTION AND SELECTIVE RESONANT TUNNELING MODEL

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ABSTRACT

Two principles of the selective resonant tunneling model are recapitulated, and applied to the case of tritium production. The model can explain the tritium production in condensed matter nuclear reaction with no neutron and gamma radiation semi-quantitatively. A similar model may excess heat with no commensurate neutron and gamma radiation. Some experiments are suggested to test our explanation.

1. TRITIUM FOUND AGAIN WITHOUT NEUTRON OR GAMMA EMISSION

A large quantity of tritium was found by W. S. Clarke[1] in the palladium cathodes from an Arata-Zhang cell[2]. Tritium was found even though neutron or gamma rays were not observed in other cold fusion experiments. However, this is not the first time tritium has been found with no strong nuclear radiation.[3-9]

Tritium has always been a strong evidence for cold fusion ever since 1989, and this has always been a challenge to skeptics who deny that cold fusion exists. Even in the report of Energy Research Advisory Board Cold Fusion Panel, scientist demanded that ‘Investigations designed to check the reported observations of excess tritium in electrolytic cells are desirable.’[10] It is quite difficult for nuclear scientists to explain how the triton was produced according to their conventional knowledge and experience, since no neutron or gamma was observed during the experiments. Many hot fusion researchers deny that the tritium itself exists, based on the lack of neutron and gamma. They believe that neutrons and gamma are the necessary products of nuclear reactions.

Can nuclear reactions happen without the emission of neutrons or gamma? The answer is positive and one of the evident examples is in the sun. H. A. Bethe proposed that the combination of two protons to form a deuteron, with positron emission:

$$H + H = D + e^{+} + \nu_e \qquad (1)$$

Bethe concluded that this ‘gives the correct order of magnitude for the sun.’ He won the Nobel Prize for this excellent work. We can see from above that not all nuclear reactions have to emit neutrons or gamma. In other words, emission of neutrons which in nature is the strong interaction between nucleons, and emission of gamma which is the electromagnetic interaction are just possible reaction channels, but they are not the necessary reaction channels. The weak interaction such as positron emission is also a kind of reaction channel as Bethe demonstrated in his paper[11].

This brings up another question: Why are neutrons or gamma are detected in almost all the well-known fusion reactions such as T(d,n)α, D(p,γ)^{4}He, and so on? The answer lies in the theory of nuclear physics. Let us take the reaction T(d,n)α as an example.

Attention should be paid to something in Fig. 1. First, the cross-section of T(d,n)α reaches the peak when the relative energy between deuterons and tritons is about 66keV(C.M. system), much less than 200keV, the maximum height of Coulomb barrier between them. When the relative energy increases from 66keV the cross-section decreases. It’s hard to explain this if tunneling through the Coulomb barrier and decay in the nuclear potential well are treated as two irrelevant processes, since it is obvious that tunneling would be easier, so that the cross-sections should be larger when relative incident energy is higher. The only reasonable explanation is that tunneling and decay processes are relevant. They are combined together as a single-step process by the bounce motion of the wave function inside the nuclear well. The two processes match at the energy of 66 keV and as a result the wave function resonates such that the deuteron and triton have the largest probability of tunneling through Coulomb potential barrier and then decay in the nuclear potential well. We can say that 66keV is the resonant energy level, and the tunneling process is in fact resonant tunneling.

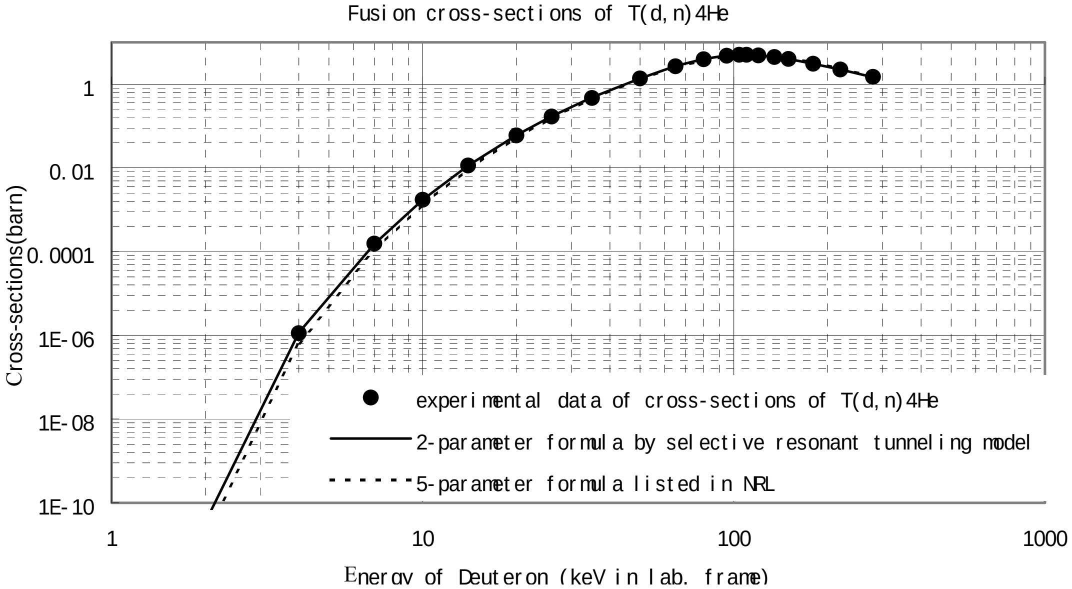

Fig1.Fusion cross-sections for reaction, T(d,n)α. The "experimental data" listed here below 8kev was extrapolated by R-matrix theory[12]. The energy is in the laboratory frame.

Selective resonant tunneling[13,14] means that tunneling through the Coulomb barrier and decay in the nuclear potential well should match not only in energy level but also match in damping (the life-time of the decay). It is easy to understand that the cross-section would not be observable if the decay is too weak and slow. However, if the decay is too strong and fast, the cross-section would not be large either, because the short lifetime of the penetrating wave may not have enough time to build up the wave amplitude in terms of constructive interference, to allow resonant tunneling.

Based on the single step selective resonant tunneling theory, it is proved that[13,14] in order to maximize the probability of reaction, the ratio of the flight time (τ_{flight}) to the lifetime of the penetrating wave inside the nuclear well (τ_{life}) must be in the order of $(1/\theta_0)^2$ for the case of incoming traveling wave.($(1/\theta_0)$ is the Gamow factor, see later).

Then, what about the case of $H+H=D+e^{+}+\nu_e$ reaction? We can see that Bethe also believed the tunneling through Coulomb barrier was a resonant tunneling because he included the irregular solution of Coulomb potential (denoted by G as a common notation) and G played an important part in his calculation. Relations between the lifetime and the flight time of the penetrating wave inside the nuclear well are also mentioned in his paper[11]. We should note that the relative incident energy is several keV in this case.

We can expect that in the case of even lower incident energy, the resonant tunneling should play an even more important role, and the lifetime of the penetrating wave should be much longer to form a resonant state.[13,14]

2. PRODUCTION OF TRITON IN PALLADIUM

Tritons may be products of the following fusion nuclear reactions

$$D + D \rightarrow p + T \tag{2}$$

$$D + p \rightarrow T + e^{+} + \nu_e \tag{3}$$

$$p + d + e \rightarrow T + \nu_e \tag{4}$$

in another form:

$$p + d \rightarrow {}^{3}He^{*} \tag{5}$$

$${}^{3}He^{*} + e \rightarrow T + \nu_e \tag{6}$$

Under electrolysis conditions like Arata-Zhang cell, protons and deuterons were absorbed in the cathodes of palladium, the relative incident energy between them is even lower than that in Bethe's calculation. In fact, the particles are in bound state (the eigen state of the lattice potential well). It can be deduced easily that resonance of wave function should be even more important to enhance the probability of the particles appearing in the nuclear well, and the matching decay rate of the penetrating wave inside the nuclear well should be much slower and weaker even than the rate of positron emission. Then the only possible reaction channel is that in equation (4). This means that protons and deuterons form a resonant state first (an excited state of Helium-3); then, Helium-3 captures an orbital electron (mostly K-capture).

In order to calculate the probability of the proton-deuteron two-body interaction, we introduce a Shrodinger equation with a Hamiltonian as follows:

$$\hat{H} = -\frac{\hbar^2}{2\mu}\nabla^2 + U(r) \tag{7}$$

$$U(r) = \begin{cases} U_1^r + iU_1^i; & (r \le r_1) \\ \frac{a}{r} + V_0; & (r_1 \le r \le r_2) \\ V_1\left[1 - \frac{R_1}{r} + \left(\frac{R_2}{r}\right)^2\right]; & (r_2 \le r \le r_3) \end{cases} \tag{8}$$

U(r) is the potential between a pair of proton and deuteron. When r < r_1 , *U(r)* is the nuclear potential well and U_1^r = -45.52Mev is the real part which can be derived from the cross-sections of the reaction p(d,^{3}He)gamma. U_1^i represents decay of wave function in the nuclear well. When r_1< r < r_2, ,*U(r)* is a shifted Coulomb barrier as that in paper [15], When r_2< r < r_3, *U(r)* is assumed in a form of Kratzer potential.r_2 =0.04098nm, r_3=0.269 nm, R_1 =0.31 nm , R_2 =0.105 nm , a=1.44* 10^{-9}eV·m,V_0=-35.14eV, V_1=3.9eV, r_0 = $a_0(A_1^{1/3} + A_2^{1/3})$, A_1 and A_2 are the mass number of deuteron and proton, respectively. a_0=1.746fm to give the correct diameter for deuteron (4.4fm)[16].

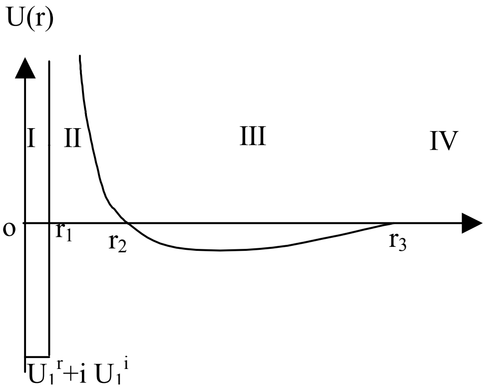

Fig. 2. Square-well model with complex potential in nuclear well region I; shielded Coulomb barrier in region II; potential well by lattice confinement effect in region

Barrier region II separates two well regions I and III. In the non-resonant case which means the depth, width and shape of the potential do not meet the resonant condition, the proton is mainly confined there and would hardly appear in region I.

$$\iiint_{I} |\psi(E,r)|^2 dr \ll \iiint_{I+II+III+IV} |\psi(E,r)|^2 dr \tag{9}$$

Then, the decay of penetrating wave inside nuclear well has a negligible effect

With a proper set of potential parameters like that in equation (8) however, reflected wave from the nuclear potential and from Coulomb barrier would interfere in region I to enhance the amplitude of the wave function. The amplitude of the wave function in region I increases gradually, and the probability that the proton will tunnel through the Coulomb barrier into the deuteron nuclear potential well is greatly enhanced.

In mathematics the solution of Shrodinger equation in region II is expressed as

$$\psi(r) = F(r) + C\,G(r) \tag{10}$$

$F(r)$ converges while $G(r)$ diverges when r—>0. C is a constant which shows the effect of the nuclear potential well. Resonance of wave function means that C is relatively large such that the probability of having proton inside the nuclear well is of the same order of magnitude as that in the whole region, i.e.:

$$\int_{\mathrm{I}} |\psi(E,r)|^2\, dr \approx \int_{\mathrm{I}+\Pi+\mathrm{III}+\mathrm{I}V} |\psi(E,r)|^2\, dr, \tag{11}$$

The decay effect, U_1^i , should be introduced here because it cannot be neglected any more and it is no longer a stationary-state problem. The solution of the Schrodinger equation, $\psi(E,r,t)$, satisfies

$$\frac{\partial |\psi(E,r,t)|^2}{\partial t} = -\nabla \cdot \vec{j} + \frac{2\,\mathrm{Im}\,U\,|\psi(E,r,t)|^2}{\hbar} \tag{12}$$

where

$$\vec{j} = \frac{i\hbar}{2\mu}(\psi \nabla \psi^* - \psi^* \nabla \psi) \tag{13}$$

$\vec{j}$ = the density of the probability current; μ=reduced mass of deuteron and proton; $\hbar$=Planck constant divided by 2π.

If we assume $\psi(E,r,t) = \psi(E,r)\exp(-iEt/\hbar)$, then the imaginary part of the potential U_1^i would imply an imaginary part of the energy, i.e., $E = E_r + iE_1^i$ and

$$E_1^i = \frac{\int \mathrm{Im}\,U\,|\psi(E,r,t)|^2\, dr}{\int |\psi(E,r,t)|^2\, dr} = \frac{U_1^i \int_{\mathrm{I}} |\psi(E,r)|^2\, dr}{\int_{\mathrm{I}+\Pi+\mathrm{III}+\mathrm{I}V} |\psi(E,r)|^2\, dr} \tag{14}$$

The decay rate of wave function in nuclear well, λ , which is also the reaction rate and a measurement of the tunneling current through the Coulomb barrier from region III to region I :

$$\lambda = E_1^i / \hbar \tag{15}$$

According to equations (14) and (15), when U_1^i approaches zero, E_1^i and λ approaches zero since no decay happens; when U_1^i approaches infinity, E_1^i and λ also approaches zero because the approximation in (11) would be no longer valid. Since the decay is so strong that the wave function deviates from the resonant state (C decreases quickly), and the probability for proton to be in region I also approaches zero. Our calculation shows that when U_1^i is about the order of 10^{-20}~10^{-24}eV, E_1^i approaches the maximum, $|E_1^i|$~ $|U_1^i|$~10^{-21}~10^{-24}eV This is in agreement with Ref.[17].

It is shown in Ref. [17] that in the bound state as discussed above, the ratio of flight time (τ_{flight}) to lifetime of the penetrating wave inside the nuclear well (τ_{life}) must be in the order of $1/\theta_0$ to see the resonant effect, i.e.

$$\tau_{\mathrm{life}} \sim \tau_{\mathrm{flight}}\theta_0 \sim 10^{-23}\theta_0 \text{ s.} \tag{16}$$

θ_0 is the barrier penetration factor obtained from the Wentzel-Kramers-Brillouin (WKB) approximation[15]:

$$\theta_0 = \exp\left[\int_{r_1}^{r_2} \frac{\sqrt{2\mu(U_2(r) - E_r)}}{\hbar} dr\right], \tag{17}$$

and θ_0 is about 10^{29}~10^{32} in our model. τ_{life} ~$1/\lambda$~$h/(4\pi E_1^i)$~ $h/(4\pi U_1^i)$~ (10^8-10^{10})s which just satisfies the matching condition between the flight time and the life-time (16).

The life-time of the penetrating particle inside the nuclear well, τ_{life}, may be roughly estimated based on the strength of the various interactions. For the strong interaction between nucleons, it is in the order of 10^{-22} -- 10^{-23} s which allows only a few oscillations bouncing back and forth motion inside the nuclear well; hence, it is too strong to allow any resonance effect at low energy. The electromagnetic interaction is weaker than the strong interaction; hence, its life-time is 10^2 -- 10^6 times longer. But this is still a strong damping to allow any observable resonant effect. For the weak interaction, there are two possible channels: β^+decay and K-capture. Usually the latter is much slower than the former for the light nucleus. We have to calculate their life-time in order to figure out which channel matches the resonant tunneling.

For the different reaction channels, their corresponding lifetime, τ_{life} are estimated in Table I:

TABLE I The life time for various interactions

Emission of nucleon	Emission of gamma	Emission of positron (^{3}He*)(β decay)	K-capture (^{3}He*)
10^{-22} --10^{-23} s	10^{-16} ~10^{-20}s	0.1~10s	10^8 ~10^{10}s

τ_{life} for the positron emission and K-capture are derived according to Fermi's theory as follows:
Based on the mass defects, we may calculate the maximum energy of positron E_0=5.475MeV, and the maximum energy of the neutrino E_ν=5.526MeV in K-capture. For the case of positron emission, the decay constant λ_β is:

$$\lambda_\beta \approx \frac{m_e^5 \cdot c^4 \cdot g^2 \cdot \left|M_{if}\right|^2}{2 \cdot \pi^3 \cdot \hbar^7} \cdot f(Z, E_0)$$

m$_e$ is the static mass of positron, g=1.415*10^{-48} J*m^3, $\left|M_{if}\right| = \left|\int \phi_i^* \cdot \phi_f \cdot \exp(-i(\vec{k}_\beta + \vec{k}_\nu) \cdot \vec{r}) d\tau\right|$ in which ϕ_i^* is the conjugated wave function of initial state of the nucleus(in our case the specially excited state of Helium-3) while ϕ_f is the wave function of the final state of the nucleus (in our case the wave function of triton). $\vec{k}_\beta$ is the wave vector of positron and $\vec{k}_\nu$ is the wave vector of neutrino $f(Z, E_0)$ is the integration of the Coulomb correction factor which is a function of charge number(Z), and the maximum energy of the charged particle. In our case $f(Z, E_0)$ =4×10^3 since Z=1, E_0 =5.475MeV. Thus, λ_β is about 1 s^{-1}

For the K-capture case the reaction rate, λ_k , is:

$$\lambda_k \approx \frac{m_e^5 \cdot c^4 \cdot g^2 \cdot |M|^2}{2 \cdot \pi^3 \cdot \hbar^7} \cdot f_k(Z, W_\nu)$$

m$_e$ is the static mass of electron, $f_k(Z', W_\nu) = 4\pi \left(\frac{Z' \cdot e^2}{\hbar \cdot c}\right)^3 \cdot W_\nu^2$. Z'=2, $W_\nu = \frac{E_\nu}{m_e \cdot c^2}$ $|M| = \left|\int \phi_i^* \cdot \phi_f \cdot \phi_K d\tau\right|$;

ϕ_K is the wave function of the K-electron. Thus, λ_k is about 1×10^{-9} s^{-1}.

It is obvious that K-capture fits the reaction process best in this case. The fact that no neutron or gamma or positron has been observed in the low energy nuclear reaction experiment also proves that our model and calculation are reasonable. Although the electrolyte in Arata-Zhang cell was originally heavy water, protons were inevitable because the cell was open to air, and the heavy water is hydrophilic in nature.

We can see that nuclear reactions in solid are different from beam-target cases that the wave function is bound state in regions I and III which makes slow reaction process such as K–capture more favorable.

The above analysis and calculations apply to the stationary state. In fact, the Hamiltonian changes slowly because the potential differs as the lattice vibrates, or with some other condition changing. Since a long period of time is needed to form the resonant state and the potential varies with time, before the resonant state is formed, C is very sensitive with the change of r_3 and U_3. That means the resonant conditions are sensitive with the change of the lattice. As we all know, the depth and width of palladium lattice vary with temperature and other conditions. We believe that this is the reason why such experiments are difficult to repeat.

According to Ref. [14], the cross-sections of nuclear reactions can be calculated by a 2-parameter formula based on the selective resonant tunneling model. U_1^r *and* U_1^i, the real part and imaginary part of the nuclear potential well, are just these two parameters. U_1^r can be obtained from the experimental values of cross-sections of nuclear reactions. For the reaction p(d,^{3}He)gamma, the U_1^r is -45.52MeV and the U_1^i is -0.038eV which is related to the decay rate through gamma radiations. The calculated values and experimental values are drawn in Fig. 2:

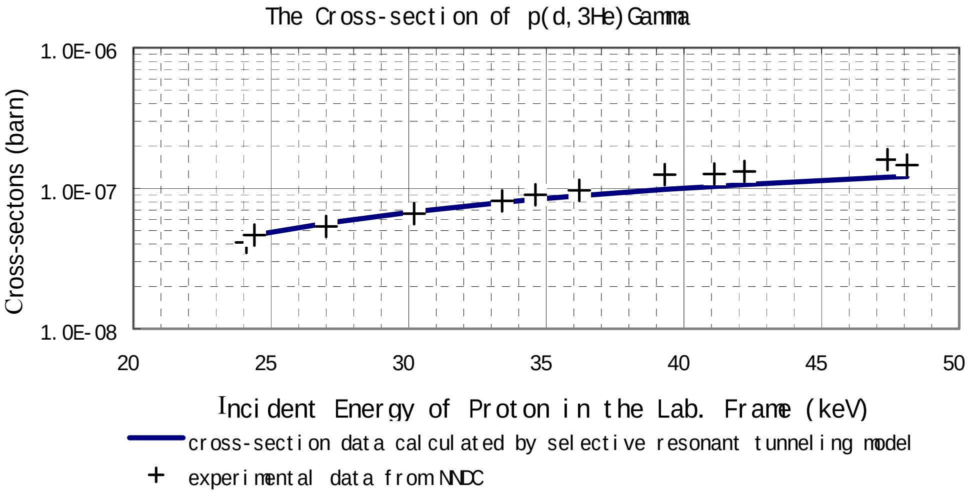

Fig. 2. The comparison of the cross-sections of 2-parameter formula with experimental values for the nuclear reaction of p(d,^{3}He)gamma

3. SOME OTHER POSSIBLE NUCLEAR TRANSMUTATIONS

As we discussed above, nuclear transmutations which were generally thought impossible under normal conditions (because the processes are so slow) are very possible in the solid state due to the resonant effect in lattice. Another probable transmutation is: ^{6}Li + p + e—>^{7}Li+ν_e which can explain the anomalous ratio of ^{7}Li to ^{6}Li observed in some experiments. In conclusion, transmutations of light and heavy nuclei are probable in solids because the weak interaction such as electron capture may match the selective resonant tunneling condition in the lattice. To verify the above hypothesis, we suggest two experiments:

1. Apply the mixture of D_2O and H_2O instead of pure D_2O in electrolytic experiment. It should be favorable to tritium production
2. Search for the sequential processes of the orbital electron capture such as the Auger X-rays and Auger electrons after transmutation of heavy nuclide.

If these two experiments have the positive results, it would lend strong support to the selective resonant tunneling model for the low energy nuclear reactions and transmutations.

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